MAT 248A, Winter 2026
Homework 0: commutative algebra review

This homework is optional

Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.

1. a) Let R be a commutative ring, and I an ideal in R. Prove that R/I is a commutative ring.
b) Prove that ideals in R/I are in bijection with ideals in R containing I.

2. Let $f = (x-1)(x-2)$ and $g = (y-2)(y-3)$. Prove that $\mathbb{C}[x,y]/(f,g)$ is a finite-dimensional vector space over $\mathbb{C}$. Find its dimension and some basis, and describe the multiplication table in this basis.

3. a) Prove that an ideal I is prime in R if and only if R/I has no zero divisors.

b) Prove that an ideal I is maximal in R if and only if R/I is a field.

4. a) Are all prime ideals maximal? Prove it or give an explicit counterexample.

b) Are all maximal ideals prime? Prove it or give an explicit counterexample.

5. Find all maximal ideals in (a) $\mathbb{C}[x]$ (b) $\mathbb{R}[x]$.

6. Assume that $\mathbf{k}$ is a finite field with q elements, and $f(x)$ an irreducible polynomial of degree d. Prove that $\mathbf{k}[x]/(f)$ is a field and compute the number of elements in it.