MAT 261A, Spring 2022
Homework 2

Due before 1:10 on Wednesday, April 13

Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.

1. The group $U(n)$ acts on $\mathbb{C}^n$ by $A(v) = Av$.

a) Find the orbit of the basis vector $e_1 = (1, 0, \ldots, 0)$ under this action.

b) Find the stabilizer of e_1 under this action.

2*. Use problem 1 and the long exact sequence of a fibration to compute the fundamental groups of $U(n)$ and $SU(n)$.

3. Let G be a matrix Lie group and G_0 the connected component of identity in G.

a) Prove that G_0 is a normal subgroup of G.

b) Prove that x and y are in the same connected component of G if and only if $x^{-1}y \in G_0$.

c) Prove that all connected components of G are homeomorphic to G_0.

Note: you can use without proof that x and y are in the same connected component of G if and only if they are connected by a path.

4. Suppose that $GL(n, \mathbb{R})$ has a continuous representation V, that is, there is a continuous homomorphism $\rho : GL(n, \mathbb{R}) \to GL(V)$. Prove that the stabilizer of any vector $v \in V$ is a matrix Lie group.