K. Takayama (Ed.)

Shock Waves

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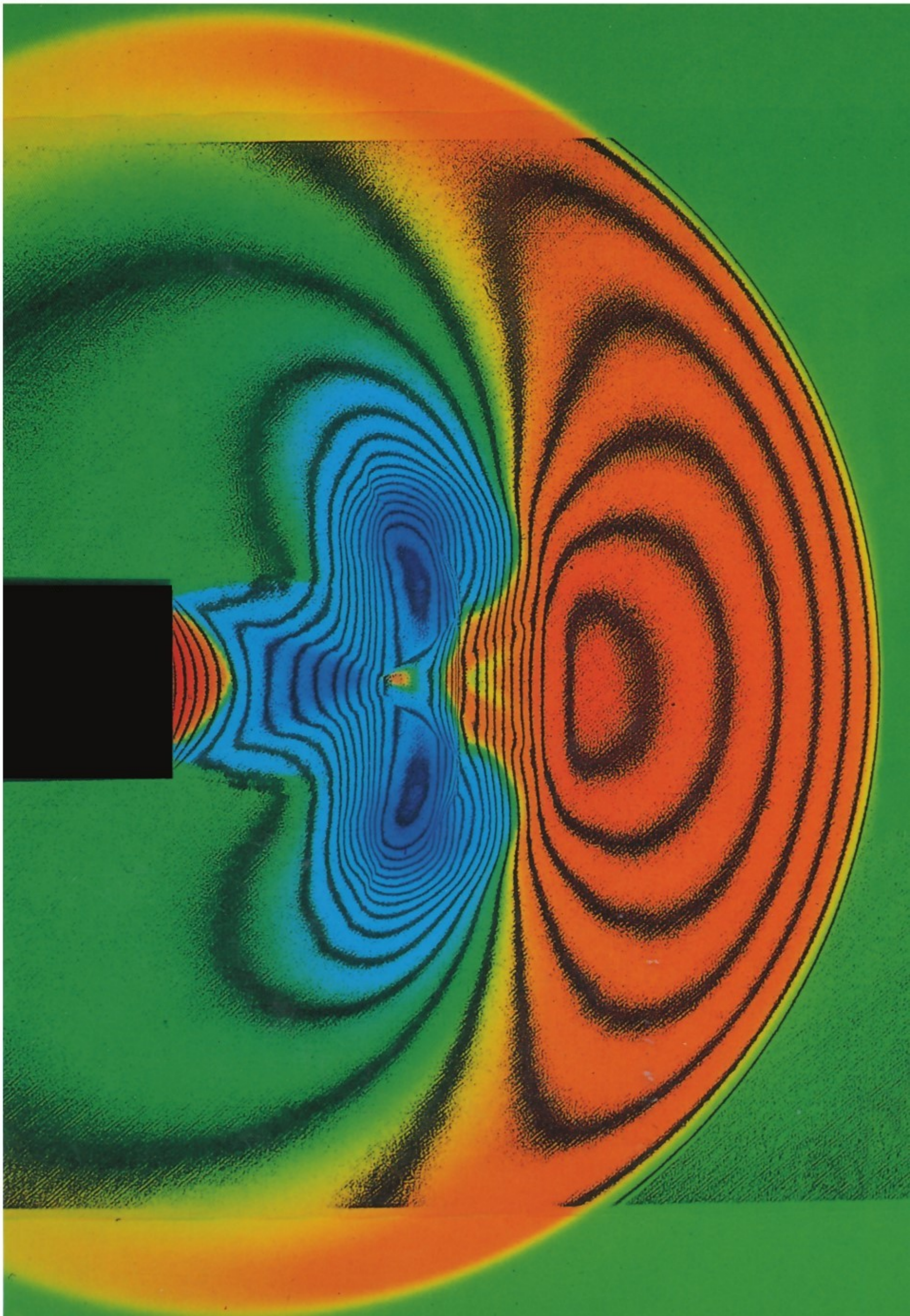

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Anomalous refraction of shock waves

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Abstract. We present the results of our numerical work on anomalous shock refraction at air/CO_2 and air/SF_6 interfaces. The numerical method was a second-order multi-fluid Godunov code. The results indicate that anomalous refraction is not one phenomena but a group of them, and their natures depend on whether the wave impedance increases or decreases during refraction.

Key words: Shock refraction, CFD, Shock impedance

1. Introduction

Refraction is expected whenever a shock passes from one material to another. Formally, it appears when the relative refractive index n is different from unity (n $\neq$ 1) during propagation, where

$$n \equiv | U_i | / | U_t | = \sin\alpha_i / \sin\alpha_t \quad . \tag{1}$$

Here U_i, U_t are the velocities of the shock in the incident and transimitting (receiving) material, and α_i, α_t are the wave angles of the shock in these materials. Nearly all refractions are accompanied by a wave that is transmitted back into the initial material. This will be a compression if the wave impedance Z increases during refraction $Z_t > Z_i$, but an expansion if Z the decreases ($Z_t < Z_i$). The wave impedance is the vector mass flux,

$$Z_i \equiv \rho_i U_i \quad ; \quad Z_t \equiv \rho_i U_t \quad , \tag{2}$$

where ρ is the material density. An alternative and equivalent definition makes use of the momentum equation,

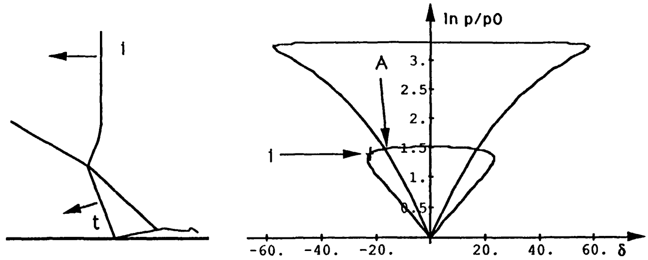
$$Z_i = \frac{P_i - P_o}{| U_{Pi} |} n_i \tag{3}$$

where P_i-P_o is the pressure jump across i, U_{Pi} is the particle (piston) velocity, and n_i is a unit vector in the direction of propagation.

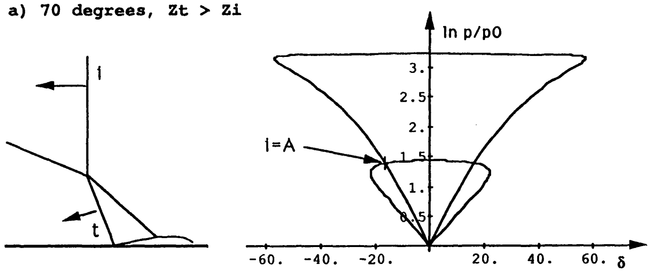
It is convenient to classify refracting systems as regular or irregular, analogous to the classificaiton of shock reflection as regular or Mach (irregular). Regular sytems have uniform flow fields between the waves but irregular sytems have at least one non-uniform region.

2. Anomalous refraction

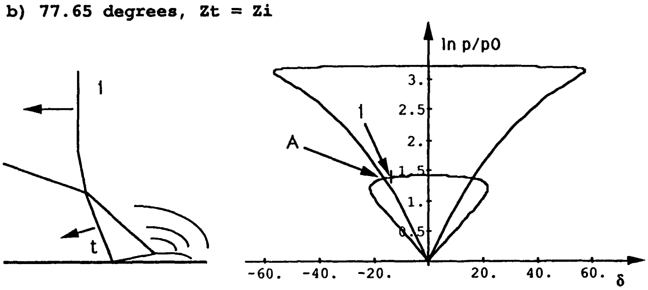
This is an irregular system first detected during the experiments of Jahn (1956). One necessary condition for its onset is that the refraction should be fast-slow $n > 1$,e.g. when a shock passes from air into CO_2 (Jahn) or from air into SF_6(Abdel-Fattah and Henderson 1978). A second condition is that the wave angle α_i of the incident shock i exceeds the condition where there is sonic flow downstream of, and relative to, i (M_1=1). This critical condition $\alpha_i = \alpha_i^*$ say is easily



a) 70 degrees, Zt > Zi



b) 77.65 degrees, Zt = Zi



c) 80 degrees, Zt < Zi

Fig. 1. Wave and polar sequence with changing wave impedance for Air/ SF_6 shock refraction ($n > 1$)

calculated from a wave diagram. The result for an arbitrary material is (Kamegai 1986; Grove and Menikoff 1989).

$$\tan^2 \alpha_i^* = \frac{U_i^2}{a_1^2 - (U_i - U_{pi})^2} \quad , \tag{4}$$

where a_1 is the speed of sound in the undisturbed incident material. For the special case of a perfect gas Eq. (4) can be more conveniently written in terms of the shock Mach number M_i and the ratio of specific heats γ

$$\tan^2 \alpha_i^* = \frac{(\gamma+1)M_i^2}{(M_i^2 - 1)[(\gamma - 1)M_i^2 + 2]} \quad . \tag{5}$$

Jahn (1956) experimented with the air/CO_2 interface for $\alpha_i > \alpha_i^*$ and obtained the type of anomalous refraction sketched in Fig.1c. The incident shock i weakens as it approaches the interface, and curves in the rearward (downstream) direction. This fact indicates that $Z_t < Z_i$. The polar diagram supports this conclusion since the pressure ratio P_i/ P_0 associated with i exceeds that associated with the equality of impedance condition ($Z_t{=}Z_i$) at the polar intersection A. In Fig.1b the polar map of i coincides with A, (i≡A) and the wave impedances of i and t are equal. Physically i is now a plane wave all the way to the inter-face. In Fig.1a, $Z_t > Z_i$ and i strengthens as it approaches the interface, and curves forward.

In Jahn's experiments it was always true that $Z_t < Z_i$ so the anomalous refraction was as illustrated in Fig.1c, but in the experiments of Abdel-Fattah and Henderson (1978) $Z_t > Z_i$ and the anomality was as in Fig.1a. When i was sufficiently strong, there was a Mach reflection in air.

3. Numerical work

We assumed that all the gases were perfect and that the flows were two-dimensional, compressible, unsteady and non-viscous. Consequently we numerically integrated the continuity and Euler equations in association with the perfect gas equations of state. The code has been described in some detail elsewhere (Colella et al. 1989; Henderson et al. 1991). Briefly it was a second order finit difference solution on a rectangular grid with reflecting boundary conditions on three sides and an inflow condition on the fourth. A second order Godunov method with operator splitting was chosen (Van Lear 1979; Colella and Woodward 1984). There was automatic refinement of the grid in regions of special interest or excessive error. This provided us with an economic method for resolving important regions of flow.

4. Results and discussion

We have computed examples of anomalous refractions at air/CO_2 and air/SF_6 interfaces. A sequence for the latter with a varying impedance relation is shown in Fig.2. In Fig.2c, $Z_t < Z_i$ and as expected, i weakens and curves in the downstream direction as it approaches the interface. In Fig.2b, $Z_t = Z_i$ and i remains everywhere a plane shock, while in Fig.2a, $Z_t > Z_i$, and i curves forward. There are some other interesting results. The t shock reflects off a rigid wall and intersects the gas mixing layer. In Fig.2c, the intersectin is a source of acoustic radiation, and this was also detected in the experiments of Abdel-Fattah and Henderson (1978). In Fig.2a, the layer shows marked instabilities, which appear to be successively linear, Kelvin-Helmholz, and chaotic.

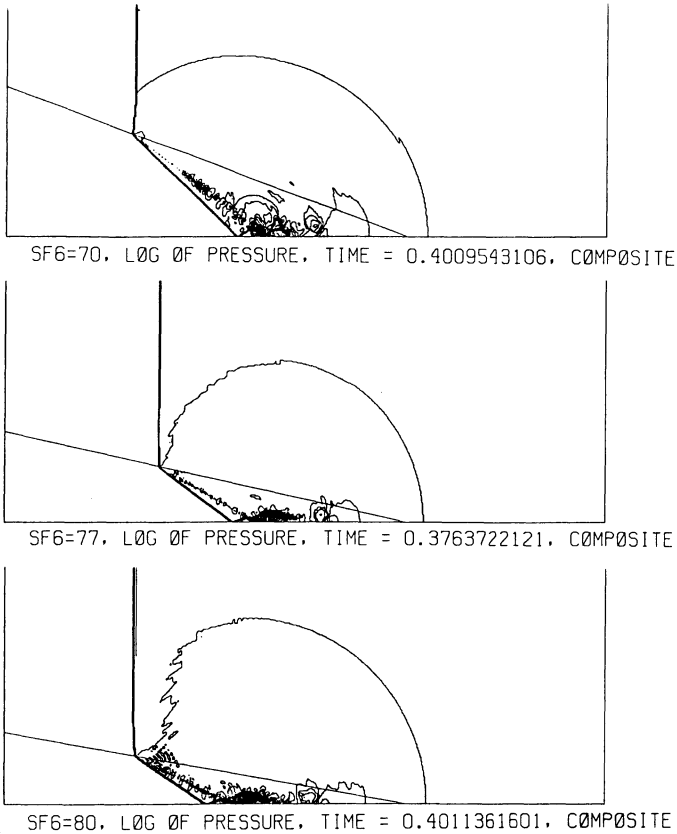


Fig. 2. Numerical contour plots for Air/SF_6 shock refraction ($n > 1$). a) $Z_t > Z_i$, b) $Z_t = Z_i$, c) $Z_t < Z_i$

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