A numerical study of shock acceleration of a diffuse helium cylinder

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Abstract: The development of a shock-accelerated diffuse helium cylindrical inhomogeneity is investigated using a new numerical method. The new algorithm is a higher-order Godunov implementation of the so-called multi-fluid equations. This system correctly models multiple-component mixtures by accounting for differential compressibility effects. This base integrator is embedded in an implementation of adaptive mesh refinement (AMR) that allows efficient increase in resolution where the computational effort is concentrated when high accuracy, or increased resolution, are required. Qualitative and quantitative comparisons with previous experimental data are excellent. The simulations show that counter-sign vortex blobs are deposited in the jet core by baroclinic generation of the curved shock wave as it traverses the jet. This vorticity deposition occurs over timescales that scale with the shock passage time ($\sim 10\ \mu s$). Three phases of development are identified and characterized. The first is the weak-deformation (WD) phase, where there is weak distortion of the helium jet due to weak vorticity-induced velocity effects. The second phase is the strong-deformation (SD) phase where there is large distortion for the jet and the vortex blobs due to large induced-velocity effects. The last is a relaxation/reorganization (RR) phase where the vorticity field reorganizes into a point-like vortex pair.

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1. Introduction

The evolution of interfaces accelerated by shock waves has been the focus of investigations for many years. This class of problem has applications in such disparate fields as inertial confinement fusion (ICF) and high-speed combustion. The literature is rich in studies on the stability of plane interfaces (Richtmyer 1960, Meshkov 1969, Zaitsev et al. 1985, and Brouillette & Sturtevant 1988). The stability of curved interfaces has received similar attention culminating with the study of Haas & Sturtevant (1987), where cylindrical or spherical shapes, constructed from soap bubbles or nitrocellulose membranes, were filled with either a light or heavy gas and then accelerated by a shock wave. This last study and the previous works relied on such membranes to separate gases. A different class of experiment on acceleration of gaseous cylinders was developed by Jacobs (1992, 1993) that was free from the effect of membranes and used a planar laser-induced fluorescence (PLIF) technique that allowed for excellent visualization of the flow and quantitative information regarding mixing.

The present simulations use a high-resolution adaptive method designed for multi-fluid flows with initial mixed layers. In this paper, the development of the helium mole fraction and vorticity field will be examined. Extensive comparisons with experimental data have also been completed as well as development of a new conceptual model describing this flow, but these cannot be described here due to space limitations.

2. Multi-fluid system and numerical method

The equations used for this study are the so-called multi-fluid equations as developed by Collela et al. (1995) and also described in Puckett & Saltzman (1992). This system is based on a volume-of-fluid approach, where the fraction of a cell volume occupied by a distinguished fluid is followed during the flow evolution. The basic assumptions in this approach are that there is pressure equilibrium within a cell, a single velocity vector for both fluids, and that all changes of state are adiabatic. The equations are given as

$$\frac{\partial f^\alpha}{\partial t} + \nabla \cdot (\boldsymbol{u} f^\alpha) = f^\alpha \frac{\hat{\Gamma}}{\Gamma^\alpha} \nabla \cdot \boldsymbol{u} \tag{1}$$

$$\frac{\partial}{\partial t}(f^\alpha \rho^\alpha) + \nabla \cdot (\boldsymbol{u} f^\alpha \rho^\alpha) = 0 \tag{2}$$

$$\frac{\partial \rho \boldsymbol{u}}{\partial t} + \nabla \cdot (\boldsymbol{u}\boldsymbol{u}\rho) + \nabla p = 0 \tag{3}$$

$$\frac{\partial}{\partial t}(f^\alpha \rho^\alpha E^\alpha) + \nabla \cdot (\boldsymbol{u} f^\alpha \rho^\alpha E^\alpha) + p f^\alpha \frac{\hat{\Gamma}}{\Gamma^\alpha} \nabla \cdot \boldsymbol{u} + f^\alpha \frac{\rho^\alpha}{\rho} \boldsymbol{u} \cdot \nabla p = 0 \tag{4}$$

where f^α, ρ^α, and E^α are the volume fraction, density, and total energy density of fluid component α. The volume fraction is defined as $f^\alpha = \Lambda_\alpha/\Lambda$ where Λ is the volume of the cell and Λ_α is the volume of the cell occupied by fluid α. Γ^α is the sound speed gamma for fluid α, and $\hat{\Gamma} = 1/\sum_\alpha (f^\alpha/\Gamma^\alpha)$ represents fraction weighted Γ for the mixture.

The pressure that appears in the above system is defined to be a thermodynamically consistent pressure and defined as $p = \sum_i \hat{\Gamma}\,(f^i p^i/\Gamma^i)$, where p^α is the partial pressure of component α. Note that the formulation is sufficiently general to allow real-gas equation of state (EOS) systems described by pressure given as a function of density and internal energy.

The solution procedure for the system is a higher-order Godunov method following that of Colella (1984) and detailed in Greenough et al. (1995). This base integrator for the multi-fluid system is embedded within an implementation of adaptive mesh refinement (AMR). This methodology is based on the original work of Berger & Oliger (1984) and later Berger & Colella (1989). This is a means for managing a refined grid hierarchy composed of logically rectangular grid patches that allows for efficient increases in resolution by focusing the computational accuracy where errors are deemed large, or in regions where high resolution is required.

3. Problem setup

The initial conditions for this problem are taken from Jacobs (1993). The shock-wave Mach number is 1.094 in air. In Budzinski (1992), a Rayleigh scattering technique is used to measure the actual molar concentrations of helium. It shows the initial profile is Gaussian in shape with a core concentration of approximately 80% helium. The initial half radius is set as $r_{\text{half}} = 0.2$, where it is defined as the distance where the helium mole fraction is half the center value. The density ratio between air and helium is $\rho_{\text{helium}}/\rho_{\text{air}} = 0.138$.

The computational domain is 9 cm wide by 27 cm long with a base grid of 36 cells wide by 108 cells long. There are two levels of refinement with the first level a factor of 4 more refined than the base grid and the second level a factor of 8 more refined than the first level of refinement. This gives an effective grid resolution of $\Delta x_{\text{fine}} = \Delta y_{\text{fine}} = 0.0078125\ cm$. The shock is initialized in air at a distance of $3r_{\text{half}}$ from the jet. All boundaries are inflow/outflow type.

4. Results and discussion

4.1. Flow visualization

Figures 1 and 2 show the evolution of the mole fraction of helium and the development of the vorticity field, respectively. Figure 1a shows the initial conditions and the initial mixed layer. At 88 μs, Fig. 1b, there is a flattening of the jet and a generally weak deformation of its original circular structure. From here out to about 400 μs, spanned in time by Figs. 1c through f, there is strong deformation of the jet as it is inverted and transformed into the vortex pair. After 400 μs, from Fig. 1g on, the flow evolves as a well-defined vortex pair. Comparing these results with those given in Jacobs (1993), the agreement is exceptional.

Based on these observations and for ease and clarity of later exposition, we define three phases of development. The first is the weak-deformation phase (WD) that occurs over short times out to about 90 μs. The second is the strong-deformation phase (SD) that occurs after WD out to about 400 μs. The last is the relaxation/reorganization (RR) phase that occurs after SD.

In the vorticity time sequence, Fig. 2a shows the shock, still essentially planar, leaving positive (in the upper half plane) vorticity behind it. By 66 μs, Fig. 2b, the shock is well out of the jet core and two counter-sign circular vortex blobs are seen located in the jet core. During the WD phase, the vortex blobs remain essentially undistorted. This is due to the fact that vorticity-induced velocity effects are very weak during the WD phase. Also, the blobs are deposited in the jet core over a timescale that scales with the shock passage time ($\sim$ 10 μs). During the SD phase however, there are large induced-velocity effects that distort and stretch the vortex blobs into something like continuous finite-thickness sheets. The underlying vortical field swirls these sheets so that they are reorganized, at later times, into a vortex pair. Note that during the SD phase, there is strong counter-sign vorticity appearing also oriented in sheets, ahead of the distorted blob. Although not shown here, this counter-sign production acts to modulate the overall circulation as it relaxes to a near constant value.

The development of this flow is seen to be determined by the interaction and evolution of the vortex blobs. Once they are present in the flow, they at first interact weakly (during WD), then interact strongly (during SD) as they change from blobs to more point-like vortices (during RR). During the course of the transformation in the vorticity field, the helium jet undergoes a similar transformation as shown. From a modeling point of view, this flow could be accurately depicted by the development of a vortex blob pair in an essentially circular helium jet (slightly flattened on the upstream side in reality). The

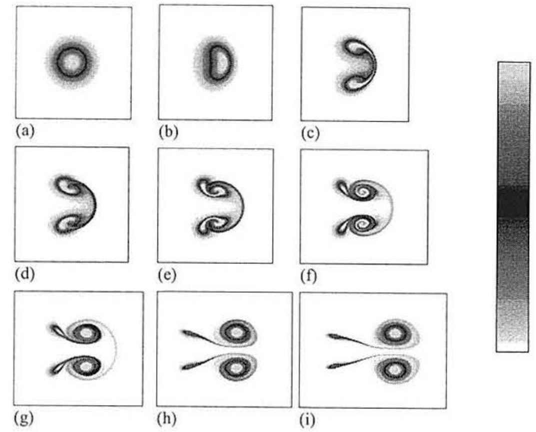

Figure 1. A sequence showing the computed helium mole-fraction evolution. Note that the shock arrival time at the jet is approximately 40 μs and all times reported are relative to the start of the calculation: (a) the initial jet, (b) 88 μs, (c) 202 μs, (d) 259 μs, (e) 318 μs, (f) 436 μs, (g) 561 μs, (h) 943 μs, (i) 1182 μs. The shock moves from left to right. The color bar [sic] to the right varies highest to lowest values from top to bottom.

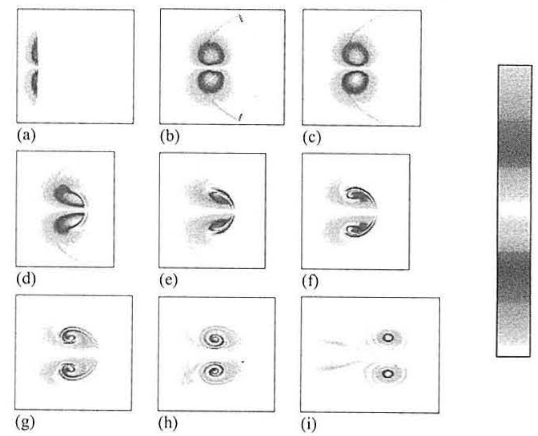

Figure 2. A sequence showing the vorticity evolution. Note that the shock arrival time at the jet is approximately 40 μs and all times reported are relative to the start of the calculation: (a) 44 μs, (b) 66 μs, (c) 88 μs, (d) 141 μs, (e) 202 μs, (f) 259 μs, (g) 318 μs, (h) 436 μs, (i) 1182 μs. The shock moves from the left to right. The color bar [sic] to the right is centered with respect to the data. Values vary highest positive to highest negative from top to bottom.

subsequent development could be predicted by their interaction and reorganization/relaxation to point vortices.

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