Overview

- As part of Research Experience for Undergraduates, we, Jianan, Aaron, Harry, and Nigeer, participated in research under the mentorship of Harsha, Dr. Ying He, and Professor Puckett.

- We tested the new active tracer particle algorithm recently implemented in the open source code ASPECT for modeling mantle convection.

- We implemented a different bilinear interpolation method that is [3], in which we designed a new algorithm to preserve the upper and lower bounds of the errors calculated on the particles in each cell.

- Used three different benchmarks to test these algorithms and the performances of all the methods. These benchmarks have finite difference [2] and finite element [1] and are steady (time independent) solutions of incompressible Stokes equations.

Motivation

- We used three different benchmarks to test these algorithms and the performances of all the methods.

- These benchmarks have finite difference [2] and finite element [1] and are steady (time independent) solutions of incompressible Stokes equations.

- We discretize the density and viscosity by initially placing the particles onto the grid. The resulting Stokes system is solved for the velocity and pressure, which is then compared to the exact solution.

The Incompressible Stokes Equations:

\[ \rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \mu \nabla^2 \mathbf{u} \]

where \( \rho \) is the density, \( \mathbf{u} \) is the velocity, \( p \) is the pressure, \( \mu \) is the viscosity, and \( \nabla \) is the vector derivative.

The Interpolation Algorithms

For each of the properties, we used two different interpolation algorithms:

- Arithmetic average:

  \[ \hat{p}(x,y) = \frac{1}{N} \sum_{i=1}^{N} p_i(x_i,y_i) \]

- Bilinear:

  \[ \hat{p}(x,y) = p_a + c_x + c_y \]

The coefficients of the bilinear function are obtained from the least squares fit of particle values and at their positions. A limiter is applied to ensure that the function value remains in the range of the maximum and minimum values on the particles. (See the algorithm described in the third column)

Algorithm to prevent overshoot and undershoot

1. We have developed a new algorithm to maintain the bounds on the particle values when interpolating to \( Q_1 \times P_0 \) elements.
2. Step 1: Find the bilinear fit for all particles.
3. Step 2: Compute max and min of all particles.
4. Step 3: Calculate the approximation at the support points using the bilinear function found in Step 1.
5. Step 4: If values at one of the support points is not between the max and min, modify the approximation at that support point to be either the max or min.
6. Step 5: Return the approximation at the support points.

The convergence rate of the error in the velocity in the \( L_2 \) norm on grid resolutions of \( h = 1/8 \) to \( 1/256 \)

\[ V \cdot \mathbf{u} = 0 \]

Errors were high when using particles with the cell average interpolation scheme.

Cell averaging is second-order accurate while bilinear interpolation is third-order accurate.

The bilinear interpolation algorithm produces errors that are indistinguishable from the direct method.

SoKz

- SoKz is a smooth solution of the steady incompressible Stokes equations.

- The density and velocity are fitted onto the grid and are solved for the velocity and pressure.

- The optimal solution can be reached because the boundary conditions for both SoKz and SoCo are free-slip boundary conditions, whereas the boundary conditions of inclusion are Dirichlet boundary conditions.

Viscosity

Benchmark Results of Active Tracer Particles In The Earth’s Mantle

J. Jiang [3], A. Kaloji [3], H. R. Levinson [3], N. Nyguen [8], H. Lokavarapu [2], Y. He [11], E. G. Puckett [1]


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Conclusions

- With enough particles and the appropriate polynomial degree for approximating the grid, our results show that the newly implemented active tracer particle algorithm converges to the exact solution of each of these benchmarks at the correct design rate.

- For both SoKz and SoCo:
  - The cell averaging method is second-order accurate and produces errors that are larger than the bilinear interpolation algorithm.
  - The higher-order interpolation method bilinear produces convergence rates that are very close to the direct method.

- Error is reduced until we run either on the grid of 512x512, at which point the error for Bilinear fail to converge to its design rate.

- Regarding SoKz benchmark, the convergence rates we obtain are only first-order, which is low compared with the other two benchmarks.

- This is due to the large viscosity jump that can not be aligned with the cell boundaries and large velocity gradient at the center of the shear flow.

- We found that in ASPECT it is not possible to reduce the error in our approximations of the solutions of SoKz and SoCo below approximately \( 10^{-7} \) to \( 10^{-11} \).

- One can see this problem in Table 3 in [1] for the two norm of velocity with the \( Q_1 \times P_0 \) element combination.

REFERENCES


