1. Let \( A = \{a_1, a_2, a_3, \ldots \} \) be an infinite set of positive integers such that for all \( i \neq j \),
\[
(a_i, a_j) = 1.
\]
Using \( A \), show that there exist infinitely many prime numbers.

Every natural number greater than 1 has a prime divisor. In particular, for every \( i \), \( a_i \) has a prime divisor, call it \( p_i \). Since \((a_i, a_j) = 1\) for all \( j \neq i \), \( p_i \) does not divide \( a_j \). Similarly, \( p_j \) does not divide \( a_i \). In particular, \( p_i \neq p_j \) for all \( i \neq j \). Thus \( \{p_i \mid i = 1, 2, \ldots \} \) is an infinite set of prime numbers. \( \square \)
2. Use the Euclidean algorithm to compute the greatest common divisor \(d = (115, 966)\). Write \(d\) as an integer linear combination of 115 and 966.

\[
\begin{align*}
966 &= 115 \cdot 8 + 46 \\
115 &= 46 \cdot 2 + 23 \\
46 &= 23 \cdot 2
\end{align*}
\]

Thus \(d = 23\). Backing out the above, we see

\[
\begin{align*}
23 &= 115 - 2 \cdot 46 \\
&= 115 - 2(966 - 8 \cdot 115) \\
&= 17 \cdot 115 - 2 \cdot 966.
\end{align*}
\]

\(\square\)