1. Show that if \( a_1, a_2, \ldots, a_n \) are pairwise relatively prime integers, then \([a_1, \ldots, a_n] = a_1 a_2 \cdots a_n\). (Remember that \([a_1, \ldots, a_n]\) is the least common multiple (LCM) of \(a_1, \ldots, a_n\), meaning \([a_1, \ldots, a_n]\) is smallest positive integer divisible by \(a_1, \ldots, a_{n-1}\) and \(a_n\). Hint: you could try to do this either using induction on \(n\), or by using prime factorizations.)

Let’s use prime factorizations. Because prime power factorizations exist, there exist distinct prime numbers \(p_1, \ldots, p_m\) such that for each \(i = 1, \ldots, n\), we can write

\[
a_i = p_1^{k_{i,1}} p_2^{k_{i,2}} \cdots p_m^{k_{i,m}},
\]

where \(k_{i,1}, \ldots, k_{i,m}\) are nonnegative integers.

Because \(a_i\) and \(a_j\) are coprime when \(i \neq j\), for all \(l = 1, \ldots, m\), there exists at most one \(i\) such that \(k_{i,l}\) is nonzero. On the other hand, by definition of the LCM,

\[
[a_1, \ldots, a_n] = p_1^{\max(k_{1,1}, \ldots, k_{n,1})} p_2^{\max(k_{1,2}, \ldots, k_{n,2})} \cdots p_m^{\max(k_{1,m}, \ldots, k_{n,m})}.
\]

Combining these two observations, we conclude that we can rearrange the \(p_l^{\max(k_{1,l}, \ldots, k_{n,l})}\) into groups so that

\[
[a_1, \ldots, a_n] = a_1 \cdots a_l.
\]
2. Find all the solutions to the following linear congruences. Describe the different congruence classes of solutions modulo 9.

(a) \[ 3x \equiv 2 \pmod{9}. \]

There are no solutions because \((3, 9) = 2 \nmid 2\).

(b) \[ 3x \equiv 6 \pmod{9}. \]

Clearly \(x = 2\) is one solution, because \(3 \cdot 2 = 6 \equiv 6 \pmod{9}\). By what we've shown in class, every other solution is of the form

\[ x = 2 + \frac{9}{(3, 9)}t = 2 + 3t, \]

where \(t\) is any integer. We've also seen that there should be \((3, 9) = 3\) congruence classes of solutions modulo 9. These are given by

\[ x \equiv 2 \pmod{9}, \]
\[ x \equiv 5 \pmod{9}, \]
\[ x \equiv 8 \pmod{9}. \]