1 Graphs of Sine and Cosine

Exercise 1
Sketch a graph of \( y = \cos(t) \). Label the multiples of \( \frac{\pi}{2} \) and \( \frac{\pi}{4} \) on your plot, as well as the amplitude and the period of the function. (Feel free to sketch the unit circle as well, and use this to guide your plot.)
Exercise 2

The following two figures were produced from an EKG (electrocardiogram) from the same patient. This is a measurement of the electrical activity in a person’s heart. The horizontal axis measures time, while the vertical axis is in millivolts. Suppose each of the thicker squares corresponds to 1/4 of a second horizontally, and 1.0 mV vertically.

1. Estimate the period and amplitude for each EKG.

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<thead>
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<th>Figure 1</th>
<th>Figure 2</th>
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<tbody>
<tr>
<td>Period</td>
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<tr>
<td>Amplitude</td>
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2. Make a conjecture about what might cause the patient’s heart to transition from the state in figure 1, to the state in figure 2.
3. Suppose the heart transitioned steadily from the state in figure 1 to the state in figure 2, in a period of 20 minutes. Find the average rate of change of the period and of the amplitude during this transition.

Exercise 3

1. Using what you know about scaling graphs, sketch a plot of the function $y = 3 \sin(x)$. What is the amplitude? What is the period?

2. Sketch a plot of the function $y = \sin(x - \frac{\pi}{2})$ by shifting your plot of $\sin(x)$ horizontally. Compare this with your plot of $y = \cos(x)$. What do you notice?
3. Sketch a plot of the function \( y = \sin(3x) \), using what you know about horizontal scaling. What are the amplitude and the period?

4. Make a conjecture about the amplitude and the period of the function

\[
y = A\sin(cx).
\]

Exercise 4

We have not yet discussed how to plot the function \( y = \tan(x) \).

1. Draw the unit circle. Find and label \( \tan \theta \) at each of the following points: \( \theta = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, 2\pi, \frac{7\pi}{4}, \frac{5\pi}{3}, \frac{11\pi}{6} \).
2. Sketch a plot of the function \( y = \tan(x) \) for \(-\pi/2 < x < \pi/2\). Then, sketch a plot of \( y = \tan(x) \) for \( \pi/2 < x < 3\pi/2 \). What do you notice?

3. What is the period of the function \( y = \tan(x) \)? What about the amplitude?

### 2 Addition Formulas

#### Exercise 5

Use the angle sum and/or difference formula to find the following.

1. \( \cos(\theta + \pi) \)

2. \( \sin(\pi - \theta) \)

3. \( \cos 3\theta \cos \theta - \sin 3\theta \sin \theta \) (use one of the formulas in reverse)
Exercise 6

1. Use the angle sum formulas for sine and cosine to prove that

\[ \tan(s + t) = \frac{\tan s + \tan t}{1 - \tan s \tan t} \]

2. Use the above, and the fact that \( \tan(-\theta) = -\tan \theta \), to prove an analogous formula for \( \tan(s - t) \).

3 Double-Angle Formulas

Exercise 7

1. If \( \sin \theta = \frac{2}{3} \) and \( \frac{\pi}{2} < \theta < \pi \), find \( \cos \theta \), \( \sin 2\theta \), \( \cos 2\theta \), and \( \cos 3\theta \).
2. Suppose \( t = 5 \cos \theta \), and \( 0 < \theta < \pi/2 \). Express \( \sin 2\theta \) in terms of \( t \).

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**Exercise 8**

1. Recall that \( \cos(2\theta) = \cos^2 \theta - \sin^2 \theta \). Rearrange this equation to solve for \( \sin^2 \theta \).

2. Now, use the identity \( \cos^2 \theta = 1 - \sin^2 \theta \) to get rid of the \( \cos^2 \theta \) term.

3. Solve for \( \sin^2 \theta \).

4. Using the formula you found above for \( \sin^2 \theta \), find \( \sin^4 \theta \).

5. Using the formula you found above for \( \sin^2 \theta \), plug in \( \theta = \frac{t}{2} \) and take the square root of both sides to prove one of the half-angle formulas.
Exercise 9

1. Suppose $\cos \theta = \frac{2}{3}$ and $0 < \theta < \pi$. Use half-angle and double-angle formulas to find $\sin \frac{\theta}{2}$ and $\cos 2\theta$.

4 Additional Recommended Exercises

8.2 1-8, 19-22, 33-36, 56-60, 63

8.3 1-8, 12-15, 41-44, 56

9.1 1-80, 83, 84

9.2 1-52, 56, 57, 60-62