1 Functions, Domain and Range, Composition

Exercise 1
For each of the following, indicate whether or not it is a function. If not, why not? If it is a function, what are the domain and range?

(a) 
\[ \begin{array}{ccc}
  a & \rightarrow & d \\
  b & \rightarrow & e \\
  c & \rightarrow & f
\end{array} \]

(b) 
\[ \begin{array}{ccc}
  a & \rightarrow & d \\
  b & \rightarrow & e \\
  c & \rightarrow & f
\end{array} \]

(c) 
\[ \begin{array}{ccc}
  a & \rightarrow & d \\
  b & \rightarrow & e \\
  c & \rightarrow & f
\end{array} \]

(d) 
\[ \begin{array}{ccc}
  a & \rightarrow & d \\
  b & \rightarrow & e \\
  c & \rightarrow & f
\end{array} \]

Exercise 2
For each function, find the domain and the range.
(a) \( f(x) = -x^2 \)  
(b) \( g(t) = \frac{1}{t+1} \)  
(c) \( q(x) = \frac{x}{x^2+1} \)  
(d) (Bonus) \( y(z) = \sqrt{\frac{z+1}{z-3}} \)

Exercise 3

Given the function, find \( f(2) \), \( f(a^2) \), and \( f(x+1) \).

(a) \( f(x) = \frac{x^2}{x+3} \)  
(b) \( f(x) = \sqrt{x - 2} \)
Exercise 4

For each of the following pairs of functions, indicate whether $f \circ g$ is a function. If so, what are the domain and the range of $f \circ g$? If not, why not? Also, what about $g \circ f$?

(a) $A\bullet B\bullet C\bullet D\bullet E\bullet F\bullet G\bullet H\bullet I$

(b) $A\bullet B\bullet C\bullet D\bullet E\bullet F\bullet G\bullet H\bullet I$

Exercise 5

Given $f$ and $g$, find $f \circ g$ and $g \circ f$. Indicate the domain and the range of $f \circ g$ and $g \circ f$.

(a) $f(x) = \frac{x+1}{x-3}$ and $g(y) = \sqrt{y-1}$. 

(b) $f(z) = \frac{1}{z^2}$ and $g(t) = (t + 1)^3$. 
Exercise 6

Suppose the radius of a spherical bubble is given by \( r = 0.15t \), where \( t \) is in seconds. How is the volume of the bubble changing in time? What is the volume after 3 seconds? (Note: the volume of a sphere is given by \( V = \frac{4}{3}\pi r^3 \) where \( r \) is the radius.)

2 Plotting Functions, Difference Quotients and Average Rate of Change

Exercise 7

Plot the following functions. Label only the \( x \)- and \( y \)-intercept(s) if they exist, and any points where the function is undefined. If there are no \( x \)- or \( y \)-intercepts, then label where the function passes through \( x = 1 \).

(a) \( f(x) = \sqrt{x + 1} \)  
(b) \( f(x) = \frac{1}{x^2} \)
(c) \( f(x) = \begin{cases} \sqrt{1-x} & \text{if } x < 1 \\ |x| & \text{if } x \geq 1 \end{cases} \)

(d) \( f(x) = \begin{cases} -x^3 & \text{if } -1 \leq x \leq 1 \\ x - 2 & \text{if } x > 1 \end{cases} \)

Exercise 8

Find the average rate of change of the given function on the interval.

(a) \( f(x) = x^2 \) on \([0, 4]\)

(c) \( f(x) = \frac{x-1}{x+2} \) on \([-1, 1]\)

(b) \( f(x) = \frac{1}{x} \) on \([\frac{1}{2}, 2]\)

(d) \( f(x) = \frac{x}{x^2 - 8} \) on \([-1, 0]\)
Exercise 9

Evaluate the given difference quotient for the given function.

(a) \( \frac{f(x) - f(a)}{x - a} \) for \( f(x) = x^2 + 3x \).

(b) \( \frac{f(x+h) - f(x)}{h} \) for \( f(x) = \frac{2}{x} \).

3 Additional Recommended Exercises

3.1 5-26, 29-38, 43, 44, 60, 61, 66, 67

3.2 17-30, 33, 34

3.3 3-6, 9-17, 21-30, 41-44

3.5 1-20, 23-31