1 Translation, Reflection, and Scaling

Exercise 1

Consider the function $y = x^2$.

(a) Sketch a plot of the function.

(b) Now, sketch a plot of the same function shifted 2 units to the right. What is the equation that describes the new function you have graphed?

(c) Sketch a plot of your function from part (b), shifted 2 units down. What is the equation that describes this plot?
Consider the function $y = 3x + 1$.

(a) Sketch a plot of the function.

(b) Now sketch a plot of the function reflected across the $y$-axis. What is the equation that describes this plot?

(c) Finally, sketch a plot of the original function $y = 3x + 1$ reflected across the origin. What is the equation that describes this plot?
Exercise 3

Consider the function \( y = \frac{1}{x} \).

(a) Sketch a plot of the function.

(b) Sketch a plot of the same function scaled by a factor of 2 vertically. What is the equation that describes this plot?

(c) Sketch a plot of the original function \( y = \frac{1}{x} \) scaled by a factor of 3 horizontally. What is the equation that describes this plot?
Exercise 4

For each of the following plots, what function is being plotted?

(a) \[ y = (x - 2)^2 + 1 \]

(c) \[ y = -(x+1)^3 \]

(b) \[ y = \frac{1}{x} - 1 \]

(d) (hint: this is a shifted version of the function \( y = \sqrt{x} \).) \[ y = \sqrt{x} - 2 \]
Exercise 5

Graph each function by applying the rules of transformation (translating, reflecting, and/or scaling) to a simpler function. For each plot, what is the simpler function that you are transforming?

(a) $f(x) = (x - 2)^2 + 3$

Simpler function: $y = x^2$

(b) $g(t) = \sqrt{-2t} + 1$

Simpler function: $y = \sqrt{x}$

(c) $h(y) = \frac{1}{y-4} - 2$

Simpler function: $h(y) = \frac{1}{y}$

(d) $z(x) = -\frac{1}{2}(x + 2)$

Simpler function: $y = x$
2 Inverse Functions

Exercise 6

Is the function one-to-one? If so, find its inverse. If not, why not?

(a) \( f(x) = \sqrt{-x+2} \)

Yes, \( y = \sqrt{-x+2} \)

\( y^2 = -x + 2 \)

\( x = 2 - y^2 \)

\( f^{-1}(y) = 2 - y^2 \)

(b) \( f(x) = \frac{1}{x-3} \)

Yes, \( y = \frac{1}{x-3} \)

\( x - 3 = \frac{1}{y} \)

\( x = \frac{1}{y} + 3 \)

\( f^{-1}(y) = \frac{1}{y} + 3 \)

(c) \( f(x) = (2x+1)^3 \)

Yes,

\( y = (2x+1)^3 \)

\( 2x+1 = \sqrt[3]{y} \)

\( 2x = \frac{\sqrt[3]{y} - 1}{2} \)

\( x = \frac{\sqrt[3]{y} - 1}{2} \)

(d) \( f(x) = 2x^2 + 1 \)

No. \( f(1) = f(-1) \).

Not one-to-one \( \Rightarrow \) no inverse function.

Exercise 7

For each of the one-to-one functions in the previous exercise, plot the function and its inverse.

(a) \( f(x) = \sqrt{-x+2} \)

\( f^{-1}(y) \)

\( f^{-1}(y) \)

\( f^{-1}(y) \)
(b) \( f(x) = \frac{1}{\sqrt{x^3}} \)

(c) \( f(x) = (2x + 1)^3 \)

(d) \( f(x) = \frac{1}{\sqrt{x^4}} + 1 \)

\*not in prev. problem
Do you notice anything interesting geometrically about the plots? Why do you think this is? (Hint: what would reflection across the line $y = x$ look like?)

Inverse function = reflection of original function across the line $y = x$.

3 Additional Recommended Exercises

3.4 1-40, 63, 64
3.5 1-20, 23-31
3.6 1-15, 18-28, 36, 37, 39-45