MAT21C—Self-Help “Second Drill Exam”—

INTRODUCTION. Do not turn to the attached copy of ‘a modified blend of old exams’ until you feel that you are close to ready to take our second exam. All of the problems on this drill exam were on previously given exams for MAT21C, though the presentation may have been varied to better reflect the way analogous questions would be asked on an exam prepared for giving this quarter.

A Self-Help Drill Exam is NOT A SAMPLE EXAM: It is just an example of what constituted an exam at an earlier time when I was making up an exam on the relevant material. YOUR ACTUAL EXAM MAY BEAR LITTLE RESEMBLANCE TO THE ONE THAT IS ATTACHED HERE. You want to come to the exam with a good, working understanding of the material related to Sections 15.1 through 15.7 in addition to any mathematics that is prerequisite to that material. Our view of the material comes from exposure to the perspectives from the text, in class discussion, class handouts, self-help worksheets, office hour participation, discussion sets, discussion sessions, etc. You should not expect to see any previously done homework problems on the exam. REMEMBER that reading the directions is important; justifications can be brief, but should be in complete sentences.

Please take a few moments to reflect upon the summary of Chapter 15 that is offered in our text. It may help focus your thoughts and give you a better sense of how things fit together.

• The Key Facts Section on page 931 of our text lists the basic formulas that are associated with the applications of double and triple integrals on which Chapter 15 has focused. For this exam, you need to know only the formulas for the volume, mass, the center of mass, and the moments about the various planes (or, in \( \mathbb{R}^2 \), about the axes). The relations not explicitly written in the box at the bottom of that page are \( \rho^2 = x^2 + y^2 + z^2 \) (relating rectangular and spherical) and \( \rho^2 = r^2 + z^2 \) (relating cylindrical and spherical). Remember that the key step in using any of the applications is describing the region or surface about which the problem is concerned. Every part must be in terms of the same coordinate system and, when translated to a specific integral, the outermost integral must have limits of integration that are constants.

• Nice renditions of the sample units can be found in Figure 10 on page 920 and in Figure 2 on page 932 (or Figure 10 on page 924); you need to know the various translations for \( dV \).

<table>
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<th>Coordinate System</th>
<th>( dV )</th>
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| rectangular       | \( dx
dy
dz \) (or \( dy
dz
dx \) or \( dz
dx
dy \); etc.) |
| cylindrical       | \( r
dr
dz
d\theta \) or \( r
dz
dr
d\theta \) |
| spherical         | \( \rho^2
d\sin\phi
d\phi
d\phi
d\theta \) or \( \rho^2
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d\theta \) |
**Directions for Use of a Self-Help Drill Exam.** The general idea is for you to take the ‘modification old exams’ under exam conditions, followed by self-grading of your work. An annotated key is offered in a separate file so that you will be able to grade your own work on the old exam. If you have a study group or partner, it can be helpful to trade off on the grading. Since everyone has different talents and rates of problem completion, don’t be surprised or concerned if you do not complete the old exam in the 50 minutes to which you limit yourself. Your goal is to gain a better understanding of how well you know some of the material, thus far, and to gain some information concerning the types of errors that you make when trying to work exam problems related to the material of emphasis. Also, your overall performance can give you some insights concerning how best to use your time during the actual exam.

**STEPS TO FOLLOW**

1. When you feel close to ready to take the exam, allow yourself about 1.5 hours (50 minutes to take the old exam plus 20 minutes to work on problems that you did not get to during the initial 50 minute block plus 20 minutes for self-grading of your work).

2. Try to keep track of the problems that you worked on after the 50 minute block was up; if you did better on some of those problems than on the others, this might give you some ideas for how you should organize your approach to taking the actual exam on Wednesday, 5/9.

3. During the first 50 minutes, work on the self-help drill exam, as much as possible, under close-to-exam conditions. **DO NOT USE NOTES, CALCULATORS, TEXTS OR CONSULTATION WITH ANYONE** while you are “taking” the second self-help drill exam.

4. After taking the drill exam under exam conditions, use the annotated key to grade your own work. **Doing the grading is very important to the usefulness of this activity**, so please give it a good try. When you have made an error that is not described in the grading comments, make your best guess to classify the type of error. As a general guideline, for simple algebraic and arithmetic errors, take off one point each; for failure to follow the directions, like providing unjustified answers or not using the approach that was specified in the directions, give yourself zero.

5. Take a moment to reflect on the types of errors that you made. Were some due to working too quickly (dropping minus signs, leaving off constants, substituting limits of integration in the wrong order, leaving off powers of functions that were used for parts of the problem, etc.)? Were some of them due to your not knowing definitions, the key base formulas (volume, mass, moment (about an axis for $\mathbb{R}^2$ or a plane for $\mathbb{R}^3$) and the center of mass) or the terms that you
can use for brief justifications of your work? KEEP IN MIND THAT “IT IS OBVIOUS” IS NEVER AN ACCEPTABLE JUSTIFICATION.

If the answer to the first question is yes, then slow down a bit and review your work to check for such things: You don’t need to complete the whole exam in order to do well, but you want to do what you do as correctly as possible.

If the answer to the second question is yes, then review the important definitions, terminology (names of various applications of the integral, the formulas for the relationships among the different coordinate systems, etc.), and/or the template (base) formulas for the applications of the integral that we have studied. Be careful to try writing definitions or formulas, separately, without looking and then compare what you did to the actual. The definition of the integral that you are expected to know is on a yellow sheet that was distributed in class; you need know only the statement that was given between two horizontal lines which is the same as what was sent via email.

For Recording Your Scores

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REMEMBER THAT DOING “WHAT YOU DO WELL” CAN BE MORE BENEFICIAL THAN MADLY, OR CARELESSLY, RACING THROUGH EVERYTHING.

Part I: Instructions. Take the following self-help drill exam under mimicked exam conditions. Sometimes it helps to have a noisy timer running during this part of the exercise. Since there is not enough space provided for you to work the problems, you should have 7-9 sheets of paper on hand. Work out one problem per page. Do not bother to copy the problems over because that will take too much time. On our actual exam, space will be provided for you to show your work. Before you start, look at the array of problems: If you don’t think that you can complete the whole exam in 50 minutes decide on a “plan of attack” that will maximize the benefit gained from the time that you spend. Then, after the 50 minutes is up, you might work on the other problems to see how well you can do them. (If you do better on the problems that you wouldn’t have gotten to than on some of the ones you chose, then you might want to think about how you went about making the choices that you made.)
I. (A) (10 points) Draw the solid lying between the planes \( z = 0 \) and \( z = 4 \) whose cross sections, in polar coordinates, for fixed \( z \) are described by

\[
2 \leq r \leq 4 \land -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}.
\]

(B) (12 points) Find the volume of the solid described in part (A).

II. (25 points) Find the center of mass for the lamina \( R \) that is bounded by \( x = 0 \), \( y = x \), and \( y = 1 \) if the density at a point \( P = (x, y) \) in \( R \) is given by \( \sigma(x, y) = 3xy^2 \). Briefly justify your work.

III. (15 points) In spherical coordinates, SET-UP \( \int_S zdV \) where \( S \) is the solid bounded above by \( x^2 + y^2 + z^2 = 49 \) below by \( z = 3 \) and laterally by the upper branch of the cone \( z^2 = 3(x^2 + y^2) \). Carefully justify your set-up. DO NOT EVALUATE the integral.
IV. (14 points) TRUE or FALSE. State your position on the line segment that is provided and briefly justify it.

(a) If the density of a lamina $R$ at a point $P = (x, y)$ is given by $f(P)$ and $R_1, R_2, ..., R_n$ is a partition of $R$ into smaller regions, the mass of $R$ is

$$\sum_{i=1}^{n} f(P_i) A_i$$

where $A_i$ is the area of $R_i$ and $P_i$ is a point in $R_i$ for each $i$.

(b) An iterated integral that is equivalent to

$$\int_{0}^{\pi/2} \int_{0}^{\pi/2} \int_{0}^{\sqrt{1-x^2}} xdzdydx$$

is

$$\int_{0}^{\pi/2} \int_{0}^{\pi/2} \int_{0}^{\cot^{-1}(\sin \theta)} \rho^3 \cos \theta \sin^2 \phi \, d\phi d\rho d\theta$$

where $(\rho, \theta, \phi)$ is a point in spherical coordinates.

V. Each of the following is a request for SET-UP of an integral. DO NOT EVALUATE THE INTEGRAL that you offer; however, the given integral should be in a form for which all that remains to be carried out is the process of integration.

(A) (12 points) A lamina $R$ is the upper half of the region that lies outside $r = 1$ and inside $r = 2 + \cos \theta$. If the density at the point $P = (x, y)$ is
given by \( f(x, y) = xy \), SET UP the integral (in proper form) that would give the moment of \( R \) about the \( y \)-axis. Carefully show the work that leads to your set-up.

(B) (12 points) Let \( S \) be the solid lying in the first octant which is bounded by \( z = 0, y = 0, y = 1, x = 1, \) and \( z = x \). If the density at \( (x, y, z) \) in \( S \) is given by \( e^{x^2} \), SET-UP an integral that would give the moment of \( S \) relative to the \( xz \)-plane. The top and bottom of the solid without the sides shaded is shown in the first figure while a complete drawing is in the second figure.