Consequently,

$$< (G_\varepsilon * f)^2 >=< f^2 > - \int_0^\varepsilon < \nabla G_t * f \cdot \nabla G_t * f > dt. \tag{B.21}$$

Since $\mathbf{F}$ is measure-preserving and will not change the integrals, both the left and right sides of (B.21) are continuous in ε.

Thus, we have

$$< (\mathcal{P}^\varepsilon_{n+1} * f)^2 >=< (\mathcal{P}^\varepsilon_n * f)^2 > - \int_0^\varepsilon < \nabla \mathcal{P}^\varepsilon_{n+t/\varepsilon} * f \cdot \nabla \mathcal{P}^\varepsilon_{n+t/\varepsilon} * f > dt \tag{B.22}$$

and if we iterate the identity, we obtain

$$< (\mathcal{P}^\varepsilon_{n+1} * f)^2 >=< f^2 > - \int_0^{(n+1)\varepsilon} < \nabla \mathcal{P}^\varepsilon_{t/\varepsilon} * f \cdot \mathcal{P}^\varepsilon_{t/\varepsilon} * f > dt. \tag{B.23}$$

With the help of Nash's inequality (A.11) and the duality argument, we again have

$$\|\mathcal{P}^\varepsilon_n - \langle\cdot\rangle\|_{1\to 2} \leq \frac{1}{c}(\varepsilon n)^{-\frac{d}{4}}, \ \forall n \in \mathbf{N} \tag{B.24}$$

$$\|\mathcal{P}^{\varepsilon *}_n - \langle\cdot\rangle\|_{1\to 2} \leq \frac{1}{c}(\varepsilon n)^{-\frac{d}{4}}, \ \forall n \in \mathbf{N}. \tag{B.25}$$

The rest of the proof goes through without modification.

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