General second-order autonomous equations

In general, a second-order autonomous equation has

- one independent variable and
- one dependent variable.

It has the form \( \frac{d^2y}{dt^2} = f \left( y, \frac{dy}{dt} \right) \).

**Example.** Simple mass-spring system

Hooke’s Law: The restoring force of the spring is proportional to the displacement from its rest position.

Using Newton’s law \( F = ma \), we get

\[ \frac{d^2y}{dt^2} = -ky \]

Let’s consider the special case where \( k = m \). We get \( \frac{d^2y}{dt^2} = -y \), and we can guess some solutions to this equation:
General 2D first-order autonomous systems

In general, a 2D first-order autonomous system of ordinary differential equations has

- one independent variable and
- two dependent variables.
- The independent variable does not appear on the right-hand sides of the differential equations.

**Example.** Recall the predator-prey systems

\[
\frac{dR}{dt} = aR - bRF \\
\frac{dF}{dt} = -cF + dRF.
\]

Let’s go through some terminology:

- initial condition:
- solution to an initial-value problem:

![Graph of predator-prey system]

The solution shown above corresponds to the initial condition \((R_0, F_0) = (1, 0.5)\) with parameter values \(a = 2\), \(b = 1.2\), \(c = 1\), and \(d = 0.9\). See the web site for the entire animation and for a related 3D animation. DETools also has a tool called PredatorPrey.
• component graphs:

• phase plane:

• solution curve in the phase plane:

• equilibrium solutions:

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- phase portrait:

\[ t = 7.58 \]

One skill that you will learn is how to make a rough sketch of the component graphs from the solution curve. There is a tool on your CD called DESketchPad which will help you practice.