Analytic Techniques:

There are few analytic techniques that work for both linear and nonlinear systems.

1. You can always check to see if a given function is a solution (no wrong answers).

For example, consider the initial-value problem

\[
\begin{align*}
\frac{dx}{dt} &= 2y - x, \\
\frac{dy}{dt} &= y
\end{align*}
\]

\((x_0, y_0) = (2, 1)\).

Using the vector notation

\[
\mathbf{Y}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}
\]

we can write this initial-value problem as

\[
\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 2y - x \\ y \end{pmatrix}, \quad \mathbf{Y}(0) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.
\]

First, let’s see what the solution looks like when we graph it with HPGSystemSolver:
Claim: The function

\[ Y(t) = \begin{pmatrix} e^t + e^{-t} \\ e^t \end{pmatrix} \]

solves the initial-value problem.

2. General solution of a partially-decoupled system

**Example.** Consider the previous system

\[
\begin{align*}
\frac{dx}{dt} &= 2y - x \\
\frac{dy}{dt} &= y.
\end{align*}
\]

We can calculate the general solution using methods we learned for first-order equations:

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Damped Harmonic Oscillator

Let’s return to our mass-spring system and add a term that models damping.

Assumption: The damping force is proportional to the speed of the mass and it acts as a restoring force.
The second-order equation
\[ m \frac{d^2 y}{dt^2} + b \frac{dy}{dt} + ky = 0 \]
and its equivalent system
\[
\begin{align*}
\frac{dy}{dt} &= v \\
\frac{dv}{dt} &= -\frac{k}{m}y - \frac{b}{m}v
\end{align*}
\]
appear in many applications. On the CD, you will find it in MassSpring and RLCCircuits, and it has also been used to study biological processes such as the blood glucose regulatory system in humans.

There is a guessing technique for the damped harmonic oscillator
\[ m \frac{d^2 y}{dt^2} + b \frac{dy}{dt} + ky = 0. \]
Example. Consider the harmonic oscillator

\[
\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 0.
\]

Its characteristic equation is

Let’s plot these solutions with HPGSystemSolver. What are the corresponding solution curves and component graphs?