More general comments

At the end of last class, I started to make some general comments about first-order differential equations, and I want to continue with those comments now.

1. What does it mean to solve the initial-value problem

\[ \frac{dy}{dt} = f(t, y), \quad y(t_0) = y_0? \]

A solution to the initial-value problem is a differentiable function \( y(t) \) defined on some interval \( a < t_0 < b \) containing \( t_0 \) such that

(a) \( y(t_0) = y_0 \) and

(b) \( \frac{dy}{dt} = f(t, y(t)) \) for all \( t \) in the interval \( a < t < b \).

2. Be careful about notation: The distinction between the independent and the dependent variables is important.

**Example 1.** \( \frac{dy}{dt} = kt \)

The solutions to this equation are \( y(t) = \frac{k^2}{2} + c \), where \( c \) is an arbitrary constant.

**Example 2.** \( \frac{dy}{dt} = ky \)

The solutions to this equation are \( y(t) = y_0e^{kt} \), where \( y_0 \) is an arbitrary constant.

3. What does the term general solution mean?
4. You should never get a wrong answer in this course:
5. Even relatively simple looking differential equations can have solutions that cannot be expressed in terms of functions that we already know and love.

Consider the initial-value problem

\[ \frac{dy}{dt} = y^3 + t^2, \quad y(0) = 0. \]

Here is the graph of the solution as generated by HPGSolver.

![Graph of the solution](image)

Our general approach in this course:

We will study differential equations
1. using the theory and
2. various techniques:
   (a) analytic techniques
   (b) geometric/qualitative techniques, and
   (c) numerical techniques.
Separable Differential Equations (an analytic technique)

First let’s recall the method of substitution for calculating integrals (really antiderivatives):
A differential equation

\[ \frac{dy}{dt} = f(t, y) \]

is **separable** if it can be written in the form

\[ \frac{dy}{dt} = \]

**Two Examples:**

1. \( \frac{dy}{dt} = -2ty^2 \)

2. \( \frac{dy}{dt} = y^3 + t^2 \)
Let’s go back to the first example

Example. \( \frac{dy}{dt} = -2ty^2 \)
We turn to FirstOrderExamples to get a sense of the graphs of these solutions:

\[
\begin{array}{c}
 y \\
 \hline
 t
\end{array}
\]

What’s the general solution to \( \frac{dy}{dt} = -2ty^2 \)? (Think before you answer.)