Homework 4-01

due 16 in class

Problem 1. Find the measure of the angles of the triangle whose vertices are $A = (-1,0)$, $B = (2,1)$, and $C = (1,-2)$. You may keep the $\cos^{-1}$ in your answer unless it is a special angle.

Solution. Let $\alpha$ be the angle with vertex $A$. Let $\mathbf{b}, \mathbf{c}$ be the vectors $\mathbf{AB}, \mathbf{AC}$, respectively. Then $\mathbf{b} = \langle 2, -2 \rangle$, $\mathbf{c} = \langle 3, 1 \rangle$. So $|\mathbf{b}| = 2\sqrt{2}$, and $|\mathbf{c}| = \sqrt{10}$, and $\mathbf{b} \cdot \mathbf{c} = 4$. Therefore,

$$\alpha = \cos^{-1} \left( \frac{\mathbf{b} \cdot \mathbf{c}}{|\mathbf{b}| |\mathbf{c}|} \right) = \cos^{-1} \left( \frac{4}{2\sqrt{20}} \right) = \cos^{-1} \frac{\sqrt{5}}{5}.$$

Similar types of computation show that the angle with vertex $B$ is $\cos^{-1} \frac{3}{5}$ and the angle with vertex $C$ is $\cos^{-1} \frac{\sqrt{5}}{5}$.

Problem 2. Find the area of the parallelogram whose vertices are given by $A = (-1,2), B = (2,0), C = (7,1)$ and $D = (4,3)$.

Solution. In order to use the cross product formula for area, we need to put these vectors in 3D. Rewrite $A = (-1,2,0), B = (2,0,0), C = (7,1,0), D = (4,3,0)$. The area of the parallelogram is given by $|\mathbf{AB} \times \mathbf{AC}|$. $\mathbf{AB} = \langle 3, -2, 0 \rangle$, $\mathbf{AC} = \langle 8, -1, 0 \rangle$. Then

$$\mathbf{AB} \times \mathbf{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -2 & 0 \\ 8 & -1 & 0 \end{vmatrix} = \mathbf{i} \left| \begin{array}{cc} -2 & 0 \\ -1 & 0 \end{array} \right| - \mathbf{j} \left| \begin{array}{cc} 3 & 0 \\ 8 & 0 \end{array} \right| + \mathbf{k} \left| \begin{array}{cc} 3 & -2 \\ 8 & -1 \end{array} \right| = 13 \mathbf{k}.$$

Therefore, the area is $|\mathbf{AB} \times \mathbf{AC}| = |13\mathbf{k}| = 13$.

Problem 3. How much work does it take to slide a crate 20m along a loading dock by pulling on it with a 200-N force at an angle of 30 degrees from the horizontal? (You can find the relevant formula for work in example 7 in the book.)

Solution. The formula for work (which is a scalar) is

$$W = \mathbf{F} \cdot \mathbf{D}$$

where $\mathbf{F}$ is the force and $\mathbf{D}$ is the displacement vector.

In the current setting, the crate is sliding, so the displacement is horizontal (parallel to the x-axis), and has length 20 (in meters). Therefore, $\mathbf{D} = \langle 20, 0 \rangle$. Since $\mathbf{F}$ is applied at an angle of 30 degrees from the horizontal and has magnitude 200 (in newtons), we can solve for the special triangle to get $\mathbf{F} = \langle 100\sqrt{3}, 100 \rangle$. Therefore,

$$W = \mathbf{F} \cdot \mathbf{D} = \langle 100\sqrt{3}, 100 \rangle \cdot \langle 20, 0 \rangle = 200\sqrt{3}.$$

The unit is the joules.