Problem 1  Give the definition of each part below.

(a) A proposition.

(b) The equivalence of two propositional forms.

(c) The intersection of two sets $A$ and $B$.

Problem 2  Show that the propositional forms $[P \Rightarrow (Q \lor R)]$ and $[(P \land \sim R) \Rightarrow Q]$ are equivalent.
Problem 3  In each of the following cases, first translate each symbolic expression into ordinary English statement. Then negate each expression symbolically, pushing the negation symbol as far in as possible. Finally, give a translation of each negated expression into ordinary English.

(a)  $(\exists e)\{e > 0 \land (\forall d)[d > 0 \Rightarrow (\forall x)(0 < |x - a| < d \Rightarrow |f(x) - L| < e)]\}$, where $d, e$, and $x$ are real numbers.

(b)  $(\forall a)(\forall b)(\forall c)(a \text{ divides } bc \Rightarrow (a \text{ divides } b \lor a \text{ divides } c))$, where $a, b, c$ are integers.
Problem 4  Determine whether each statement is true or false. If a statement is true, prove it. If it is false, give a counterexample.

(a) If $x$ is an even integer and $y$ is a multiple of $x$ (by an integer), then $y$ is even.

(b) Let $t$ be a real number. If $t^2 = 9$, then $t > 2$.

(c) Let $x$ and $y$ be integers. If $xy$ is odd, then both $x$ and $y$ are odd.
Problem 5
(a) Write the tautology that justifies the proof of a conditional proposition by contradiction. (Hint: Instead of proving $P \Rightarrow Q$, what is the equivalent propositional form we show?)

(b) Prove the following by contradiction: Let $x$ be an integer. If $x^2$ is even, then $x$ is even.

Problem 6 Let the universe be all real numbers $\mathbb{R}$. For any $B \subseteq \mathbb{R}$, define $B^* = B \cup \{0\}$. Prove that $B = B^*$ iff $0 \in B$. 
Problem 7  Let $A$, $B$ and $C$ be sets. Indicate if each statement is true or false and prove your answer:

(a) $(A - B) \cup (A - C) = A - (B \cup C)$.

(b) $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

(c) If $A \cap B \cap C = \emptyset$, then $A \cap B = A \cap C = B \cap C = \emptyset$. 