Problem 1  
Give the definition of each part below.

(a) A symmetric relation on a set $A$.

(b) A partition of a nonempty set $A$.

(c) The characteristic function of a set $A$.

Problem 2  
Give the equivalence relation $R$ on $A = \{1, 2, 3, 4\}$ such that $A/R$ is the partition $\\{\{1, 2, 3\}, \{4\}\}$. No justification is required.
Problem 3  Let $\mathcal{A} = \{A_\alpha : \alpha \in \Delta\}$ be a family of sets and let $B$ be a set. Prove that

$$B \cup \left( \bigcap_{\alpha \in \Delta} A_\alpha \right) = \bigcap_{\alpha \in \Delta} (B \cup A_\alpha).$$
Problem 4

(a) State the principle of mathematical induction (PMI).

(b) Use the PMI to prove that for any \(n \in \mathbb{N}\), \(\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}\).
Problem 5  For each part, determine whether $R$ is an equivalence relation and prove your answer. In the case when $R$ is an equivalence relation, find $\frac{T}{2}$.

(a) $R$ is the relation on $\mathbb{Q}$ given by $xRy$ iff $x = 3^k y$, for some $k \in \mathbb{N}$.

(b) $R$ is the relation on $\mathbb{Q}$ given by $xRy$ iff $x = 3^k y$, for some $k \in \mathbb{Z}$. 
Problem 6  Give a relation on $A = \{1, 2, 3\}$ that is reflexive and transitive but not symmetric. Explain why the given relation is not symmetric.

Problem 7  Let $F$ be a relation from $A$ to $B$ and $G$ a relation from $B$ to $C$. Prove that $\text{Dom}(G \circ F) \subseteq \text{Dom}(F)$.

Problem 8  Let $f : A \rightarrow B$ with $\text{Rng}(f) = C$. Prove that if $f^{-1}$ is a function, then $f^{-1} \circ f = I_A$. 
Problem 9 Suppose $f : A \to B$ and $g : C \to D$. Let $E = \{x \mid f(x) = g(x)\}$. In class, we proved that $f \cap g$ is a function with domain $E$ by showing that $f \cap g = g|_E$. Prove this result directly by verifying the two conditions in the definition of a function.