Section 2.2

[1] 
(a) \( f(x) = x^2 \). Using power rule we have \( f'(x) = 2x \). Therefore slope of the tangent at \( x = 1 \) is \( f'(1) = 2 \).

(b) \( f(x) = x^{1/2} \). Using power rule we have \( f'(x) = \frac{1}{2}x^{-1/2} \). Therefore slope of the tangent at \( x = 1 \) is \( f'(1) = \frac{1}{2} \).

[4] 
(a) \( f(x) = x^{-1/2} \). Using power rule we have \( f'(x) = -\frac{1}{2}x^{-3/2} \). Therefore slope of the tangent at \( x = 1 \) is \( f'(1) = -\frac{1}{2} \).

(b) \( f(x) = x^{-2} \). Using power rule we have \( f'(x) = -2x^{-3} \). Therefore slope of the tangent at \( x = 1 \) is \( f'(1) = -2 \).

[6] \( f(x) = -2 \). Using constant rule we have \( f'(x) = 0 \).

[12] \( f(x) = x^3 - 9x^2 + 2 \). Using sum-difference rule, constant multiple rule, power rule, and constant rule we have

\[
f'(x) = \frac{d}{dx}[x^3] - \frac{d}{dx}[9x^2] + \frac{d}{dx}[2] \\
= 3x^2 - 9\frac{d}{dx}[x^2] + 0 \\
= 3x^2 - 18x
\]

[15] \( f(t) = 4t^{4/3} \). Using power rule, and constant multiple rule we have \( f'(t) = 4 \times \frac{4}{3}t^{4/3 - 1} = \frac{16}{3}t^{1/3} \).

[18] \( g(x) = 4\sqrt{x} + 2 \). Using sum-difference rule, constant multiple rule, constant rule, and
power rule we have

\[ g'(x) = 4 \frac{d}{dx}[\sqrt[3]{x}] + 0 \]
\[ = 4 \frac{d}{dx}[x^{1/3}] \]
\[ = \frac{4}{3}x^{1/3-1} \]
\[ = \frac{4}{3}x^{-2/3} \]

[23] \( f(x) = \frac{1}{(4x)^3} \). we rewrite it as \( f(x) = \frac{1}{4^3x^3} = \frac{1}{4^3}x^{-3} \). Using power rule we have \( f'(x) = \frac{1}{4^3} \times (-3)x^{-4} \). After simplification we have \( f'(x) = -\frac{3}{64x^4} \).

[25] \( f(x) = \frac{\sqrt{x}}{x} \). Rewrite it as \( f(x) = x^{1/2} = x^{-1/2} \). Using power rule we have \( f'(x) = -\frac{1}{2}x^{-1/2-1} = -\frac{1}{2}x^{-3/2} \). After simplification we have \( f'(x) = -\frac{1}{2\sqrt{x^3}} \).

[26] \( f(x) = \frac{4x}{x^3} \). Rewrite it as \( f(x) = 4x \times x^{-3} = 4x^4 \). Using power rule we have \( f'(x) = 4 \times 4x^3 = 16x^3 \). No further simplification is needed.

[29] \( f(x) = -\frac{1}{2}x(1 + x^2) \). Using product rule and power rule we have

\[ f'(x) = \frac{d}{dx} \left[ -\frac{x}{2} (1 + x^2) + \left( -\frac{x}{2} \right) \frac{d}{dx} [(1 + x^2)] \right] \]
\[ = -\frac{1}{2}(1 + x^2) - \frac{x}{2} (0 + 2x) \]
\[ = -\frac{1}{2}(1 + x^2) - x^2 \]
\[ = -\frac{1}{2} - \frac{3}{2}x^2. \]

Therefore value of the derivative at \((1, -1)\) is \( f'(1) = -\frac{1}{2} - \frac{3}{2} = -2 \).

[31] \( f(x) = (2x + 1)^2 \). Using power rule we have

\[ f'(x) = \frac{d}{dx} [(2x + 1)^2] \]
\[ = \frac{d}{dx} [4x^2 + 4x + 1] \]
\[ = (4 \times 2x) + 4 \]
\[ = 8x + 4. \]

Therefore the value of the derivative at \((0, 1)\) is \( f'(0) = 4 \).
This problem can also be solved using the chain rule. We will discuss about that in next lecture.

\[ f(x) = \frac{2x^3-4x^2+3}{x^2} \]. Using quotient rule we obtain

\[
\begin{align*}
  f'(x) &= \frac{x^2 \frac{d}{dx}[2x^3 - 4x^2 + 3] - (2x^3 - 4x^2 + 3) \frac{d}{dx}[x^2]}{x^4} \\
  &= \frac{x^2[6x^2 - 8x] - (2x^3 - 4x^2 + 3) \times 2x}{x^4} \\
  &= \frac{[6x^4 - 8x^3] - [4x^4 - 8x^3 + 6x]}{x^4} \\
  &= \frac{2x^4 - 6x}{x^4} \\
  &= 2 - \frac{6}{x^3}.
\end{align*}
\]

*Alternative method:* We can rewrite the function as \( f(x) = 2x - 4 + 3x^{-2} \). Now using power rule we have

\[
  f'(x) = 2 + 3 \times (-2)x^{-3} = 2 - \frac{6}{x^3}.
\]

\[ f(x) = \frac{-6x^3+3x^2-2x+1}{x} \]. We can rewrite the function as \( f(x) = -6x^2 + 3x - 2 + x^{-1} \). Using the power rule we have

\[
  f'(x) = -12x + 3 - x^{-2}.
\]

\[ f(x) = \sqrt[3]{x} + \sqrt[5]{x} \]. Using power rule we have

\[
\begin{align*}
  f'(x) &= \frac{d}{dx}[\sqrt[3]{x} + \sqrt[5]{x}] \\
  &= \frac{d}{dx}[x^{1/3} + x^{1/5}] \\
  &= \frac{1}{3}x^{-2/3} + \frac{1}{5}x^{-4/5} \\
  &= \frac{1}{3}x^{-2/3} + \frac{1}{5}x^{-4/5}.
\end{align*}
\]

Therefore slope of the tangent line is \( f'(1) = \frac{1}{3} + \frac{1}{5} = \frac{8}{15} \). Hence equation of the tangent line at \((1, 2)\) is

\[
  y - 2 = \frac{8}{15}(x - 1)
\]

\( i.e., \) \[ 15y - 30 = 8x - 8 \]

\( i.e., \) \[ 15y - 8x - 22 = 0. \]
[53] \( f(x) = \frac{1}{2} x^2 + 5x \). Using power rule we have \( f'(x) = \frac{1}{2} 2x + 5 = x + 5 \). We know that slope of the tangent line at \( x \) is \( f'(x) \). If the tangent line is horizontal, then slope of the tangent line must be zero. Solving the equation \( f'(x) = 0 \) we obtain \( x = -5 \). Therefore at \( x = -5 \) we have \( f'(x) = 0 \). Hence at \( x = -5 \) the graph of \( f(x) \) has a horizontal tangent line.

[57(a)] Given \( f(x) = h(x) - 2 \). Differentiating both sides with respect to \( x \) we get \( f'(x) = h'(x) - 0 = h'(x) \). Since we know that \( f'(1) = 3 \), therefore we have \( h'(1) = f'(1) = 3 \).

[57(d)] Given \( h(x) = -1 + 2f(x) \). Differentiating both sides with respect to \( x \) we get \( h'(x) = 0 + 2f'(x) = 2f'(x) \). Since \( f'(1) = 3 \), we get \( h'(1) = 2f'(1) = 6 \).

Section 2.4

[5] \( g(x) = (x^2 - 4x + 3)(x - 2) \). Using product rule

\[
\begin{align*}
g'(x) &= \frac{d}{dx}[x^2 - 4x + 3](x - 2) + (x^2 - 4x + 3)\frac{d}{dx}[x - 2] \\
&= (2x - 4)(x - 2) + (x^2 - 4x + 3)(1 + 0) \\
&= (2x - 4)(x - 2) + (x^2 - 4x + 3).
\end{align*}
\]

Therefore \( g'(4) = (8 - 4)(4 - 2) + (16 - 16 + 3) = 8 + 3 = 11 \).

Alternative method: Rewrite the function as \( g(x) = (x - 1)(x - 3)(x - 2) \). Using general product rule we get

\[
\begin{align*}
g'(x) &= \left[\frac{d}{dx}(x - 1)\right](x - 3)(x - 2) + (x - 1)\left[\frac{d}{dx}(x - 3)\right](x - 2) + (x - 1)(x - 3)\left[\frac{d}{dx}(x - 2)\right] \\
&= (1 + 0)(x - 3)(x - 2) + (x - 1)(1 + 0)(x - 2) + (x - 1)(x - 3)(1 + 0) \\
&= (x - 3)(x - 2) + (x - 1)(x - 2) + (x - 1)(x - 3).
\end{align*}
\]

Therefore \( g'(4) = (4 - 3)(4 - 2) + (4 - 1)(4 - 2) + (4 - 1)(4 - 3) = 2 + 6 + 3 = 11 \).

[19] \( f(x) = \frac{4x^2 - 3x}{8\sqrt{x}} \). Rewrite the function as

\[
\begin{align*}
f(x) &= \frac{4x^2 - 3x}{8x^{1/2}} \\
&= \frac{1}{8} (4x^2 - 3x)x^{-1/2} \\
&= \frac{1}{8} (4x^{3/2} - 3x^{1/2}) \\
&= \frac{1}{2} x^{3/2} - \frac{3}{8} x^{1/2}.
\end{align*}
\]
Using power rule we obtain

\[
f'(x) = \left( \frac{1}{2} \times \frac{3}{2} x^{1/2} \right) - \left( \frac{3}{8} \times \frac{1}{2} x^{-1/2} \right) = \frac{3}{4} x^{1/2} - \frac{3}{16} x^{-1/2} = \frac{3}{4} \sqrt{x} - \frac{3}{16 \sqrt{x}}
\]

[21] \( f(x) = \frac{x^2 - 4x + 3}{x - 1} \). Note that the function is not defined for \( x = 1 \) (we have \( \frac{0}{0} \) form for \( x = 1 \)). We can rewrite the function as \( f(x) = \frac{(x-1)(x-3)}{x-1} = x - 3 \) for \( x \neq 1 \). Differentiating, we get \( f'(x) = 1 + 0 = 1 \) for \( x \neq 1 \).

[31] \( f(x) = \frac{3 - 2x - x^2}{x^2 - 1} \). This function is not defined for \( x = \pm 1 \). We can rewrite the function as \( f(x) = \frac{(1-x)(3+x)}{(x+1)(x-1)} = -\frac{3+x}{x+1} \) if \( x \neq \pm 1 \). Using quotient rule we have

\[
f'(x) = -\frac{(x+1) \left[ \frac{d}{dx} (3 + x) \right] - (3 + x) \left[ \frac{d}{dx} (x + 1) \right]}{(x + 1)^2} x + 1)^2
\]

\[
= -\frac{(x+1) - (3 + x)}{(x + 1)^2}
\]

\[
= -\frac{-2}{(x + 1)^2}
\]

\[
= \frac{2}{(x + 1)^2}
\]

for \( x \neq \pm 1 \).
[37] \( g(x) = \left( \frac{x^2 - 3}{x + 4} \right) (x^2 + 2x + 1) \). Using product rule and quotient rule we have

\[
g'(x) = \left[ \frac{d}{dx} \left( \frac{x^2 - 3}{x + 4} \right) \right] (x^2 + 2x + 1) + \left( \frac{x^2 - 3}{x + 4} \right) \left[ \frac{d}{dx} (x^2 + 2x + 1) \right]
\]

\[
= \left[ \frac{(x + 4) \frac{d}{dx} (x^2 - 3) - (x^2 - 3) \frac{d}{dx} (x + 4)}{(x + 4)^2} \right] (x^2 + 2x + 1) + \left( \frac{x^2 - 3}{x + 4} \right) [2x + 2]
\]

\[
= \left[ \frac{(x + 4) - (x - 3)}{(x + 4)^2} \right] (x^2 + 2x + 1) + 2 \left( \frac{x - 3}{x + 4} \right) (x + 1)
\]

\[
= \frac{7}{(x + 4)^2} (x^2 + 2x + 1) + 2 \left( \frac{x - 3}{x + 4} \right) (x + 1)
\]

\[
= \frac{7(x + 1)^2}{(x + 4)^2} + 2 \left( \frac{x - 3}{x + 4} \right) (x + 1)
\]

\[
= \frac{(x + 1)}{(x + 4)} \left[ \frac{7(x + 1)}{(x + 4)} + 2(x - 3) \right]
\]

\[
= \frac{(x + 1)}{(x + 4)} \left[ \frac{7(x + 1) + 2(x - 3)(x + 4)}{(x + 4)} \right]
\]

\[
= \frac{(x + 1)}{(x + 4)} \left[ \frac{7x + 7 + 2(x^2 + x - 12)}{(x + 4)} \right]
\]

\[
= \frac{(x + 1)}{(x + 4)} \left( \frac{2x^2 + 9x - 17}{(x + 4)} \right)
\]

\[
= \frac{2x^3 + 11x^2 - 8x - 17}{(x + 4)^2}
\]

[43] \( f(x) = \left( \frac{x + 5}{x - 1} \right) (2x + 1) \). Using product rule and quotient rule we have

\[
f'(x) = \left[ \frac{d}{dx} \left( \frac{x + 5}{x - 1} \right) \right] (2x + 1) + \left( \frac{x + 5}{x - 1} \right) \left[ \frac{d}{dx} (2x + 1) \right]
\]

\[
= \left[ \frac{(x - 1) \frac{d}{dx} (x + 5) - (x + 5) \frac{d}{dx} (x - 1)}{(x - 1)^2} \right] (2x + 1) + \left( \frac{x + 5}{x - 1} \right) [2 + 0]
\]

\[
= \left[ \frac{(x - 1) - (x + 5)}{(x - 1)^2} \right] (2x + 1) + 2 \left( \frac{x + 5}{x - 1} \right)
\]

\[
= \frac{-6(2x + 1)}{(x - 1)^2} + 2 \left( \frac{x + 5}{x - 1} \right).
\]
Slope of the tangent line at $x = 0$ is

$$f'(0) = \frac{-6}{(-1)^2} + 2 \frac{5}{-1}$$

$$= -6 - 10$$

$$= -16.$$ 

Therefore equation of the tangent line at $(0, -5)$ is

$$y - (-5) = -16(x - 0)$$

i.e., $y + 16x + 5 = 0.$