Section 2.5

[4] Given function \( y = (x^2 + 1)^{4/3} \). Inside function is \( u = g(x) = x^2 + 1 \), and outside function is \( y = f(u) = u^{4/3} \).

[6] Given function \( y = \sqrt{9 - x^2} \). Inside function is \( u = g(x) = 9 - x^2 \), and outside function is \( y = f(u) = \sqrt{u} \).

[14] \( f(x) = \frac{x^4 - 2x + 1}{\sqrt{x}} \). At first look, we are tempted to use quotient rule. But it can be solved efficiently using simple power rule only. Rewrite \( f(x) = (x^4 - 2x + 1)x^{-1/2} = x^{7/2} - 2x^{1/2} + x^{-1/2} \). Now using power rule we get

\[
\frac{d}{dx} f(x) = \frac{7}{2}x^{5/2} - 2 \cdot \frac{1}{2} x^{-1/2} - \frac{1}{2} x^{-3/2} = \frac{7}{2}x^{5/2} - \frac{1}{x^{1/2}} - \frac{1}{2x^{3/2}}.
\]

[Note: we can also use quotient rule. But that would take longer time].

[27] \( s(t) = \sqrt{2t^2 + 5t + 2} \). Take \( f(t) = 2t^2 + 5t + 2 \). Then we have \( s(t) = [f(t)]^{1/2} \). Using general power rule we have

\[
s'(t) = \frac{1}{2} [f(t)]^{-1/2} f'(t) = \frac{1}{2} [f(t)]^{-1/2} (4t + 5 + 0) = \frac{1}{2} (2t^2 + 5t + 2)^{-1/2} (4t + 5) = \frac{4t + 5}{2\sqrt{2t^2 + 5t + 2}}.
\]

[28] \( y = \sqrt[3]{3x^3 + 4x} \). Take \( f(x) = 3x^3 + 4x \). Then we have \( y = [f(x)]^{1/3} \). Using general
power rule we have

\[
\frac{dy}{dx} = \frac{1}{3} [f(x)]^{-2/3} f'(x)
\]

\[
= \frac{1}{3} [f(x)]^{-2/3} (9x^2 + 4)
\]

\[
= \frac{1}{3} (x^3 + 4x)^{-2/3} (9x^2 + 4)
\]

\[
= \frac{9x^2 + 4}{3(x^3 + 4x)^{2/3}}.
\]

[38] \( f(x) = x\sqrt{x^2 + 5} \). Using product rule we have

\[
f'(x) = \frac{d}{dx} [x] \sqrt{x^2 + 5} + x \frac{d}{dx} [\sqrt{x^2 + 5}]
\]

\[
= \sqrt{x^2 + 5} + x \left[ \frac{1}{2} (x^2 + 5)^{-1/2} \cdot (2x + 0) \right] \quad \text{(chain rule/general power rule)}
\]

\[
= \sqrt{x^2 + 5} + \frac{x^2}{\sqrt{x^2 + 5}}.
\]

Therefore slope of the tangent line is \( f'(2) = \sqrt{4 + 5} + \frac{4}{\sqrt{4 + 5}} = 3 + \frac{4}{3} = \frac{13}{3} \). Notice that \( f(2) = 2\sqrt{4 + 5} = 6 \). Hence equation of the tangent line is

\[
y - f(2) = f'(2)(x - 2)
\]

\[
i.e., \quad y - 6 = \frac{13}{4} (x - 2)
\]

\[
i.e., \quad 4y - 24 = 13x - 26
\]

\[
i.e., \quad 4y - 13x + 2 = 0.
\]

[61] \( f(x) = \sqrt{x^2 + 1} - \sqrt{x^2 - 1} \). Using chain rule (or general power rule) we have

\[
f'(x) = \frac{1}{2} (x^2 + 1)^{-1/2} \frac{d}{dx} [x^2 + 1] - \frac{1}{2} (x^2 - 1)^{-1/2} \frac{d}{dx} [x^2 - 1]
\]

\[
= \frac{1}{2} (x^2 + 1)^{-1/2} (2x + 0) - \frac{1}{2} (x^2 - 1)^{-1/2} (2x + 0)
\]

\[
= \frac{x}{\sqrt{x^2 + 1}} - \frac{x}{\sqrt{x^2 - 1}}.
\]
\[ y = \left( \frac{4x^2}{3-x} \right)^3. \] Using chain rule (or general power rule) we have
\[
\frac{dy}{dx} = 3 \left( \frac{4x^2}{3-x} \right)^2 \frac{d}{dx} \left[ \frac{4x^2}{3-x} \right] = 3 \left( \frac{4x^2}{3-x} \right)^2 \left[ \frac{(3-x) \frac{d}{dx}[4x^2] - 4x^2 \frac{d}{dx}[3-x]}{(3-x)^2} \right] \text{ (quotient rule)}
\]
\[
= 3 \left( \frac{4x^2}{3-x} \right)^2 \left[ \frac{8x(3-x) - 4x^2(0-1)}{(3-x)^2} \right] = 3 \left( \frac{4x^2}{3-x} \right)^2 \left[ \frac{24x - 8x^2 + 4x^2}{(3-x)^2} \right] = 3 \left( \frac{4x^2}{3-x} \right)^2 \left[ \frac{24x - 4x^2}{(3-x)^2} \right] = 3 \left( \frac{4x^2}{3-x} \right)^2 \left[ \frac{4x(6-x)}{(3-x)^2} \right] = 12x \left( \frac{4x^2}{3-x} \right)^2 \frac{6-x}{(3-x)^2}.
\]

\[
[66] s(x) = \frac{1}{\sqrt{x^2-3x+4}}. \text{ We can rewrite } s(x) = (x^2 - 3x + 4)^{-1/2}. \text{ Using chain rule (or general power rule) we have}
\]
\[
s'(x) = -\frac{1}{2}(x^2 - 3x + 4)^{-3/2} \frac{d}{dx}[x^2 - 3x + 4] = -\frac{1}{2}(x^2 - 3x + 4)^{-3/2}[2x - 3] = -\frac{2x - 3}{2(x^2 - 3x + 4)^{3/2}}.
\]

Therefore slope of the tangent line is \( s'(3) = -\frac{6-3}{2(9-9+4)^{3/2}} = -\frac{3}{2 \cdot 2^{3/2}} = -\frac{3}{16}. \) Hence equation of the tangent line at \( (3, \frac{1}{2}) \) is
\[
\begin{align*}
y - \frac{1}{2} &= -\frac{3}{16} (x - 3) \\
16y - 8 &= -3x + 9 \\
16y + 3x - 17 &= 0.
\end{align*}
\]

Section 8.4
[22] \( y = \tan e^x \). Using chain rule we have
\[
\frac{dy}{dx} = \sec^2 e^x \frac{d}{dx}[e^x] = e^x \sec^2 e^x.
\]

[24] \( y = -\sin^4 2x \). Using chain rule we have
\[
\frac{dy}{dx} = -4\sin^3 2x \frac{d}{dx} [\sin 2x] = -4\sin^3 2x \cos 2x \frac{d}{dx} [2x] = -8\sin^3 2x \cos 2x.
\]

[25] \( y = e^{2x} \sin 2x \). Using product rule and chain rule we have
\[
\frac{dy}{dx} = \frac{d}{dx}[e^{2x}] \sin 2x + e^{2x} \frac{d}{dx}[\sin 2x] = e^{2x} \frac{d}{dx} [2x] \sin 2x + e^{2x} \cos 2x \frac{d}{dx} [2x] = 2e^{2x} \sin 2x + 2e^{2x} \cos 2x = 2e^{2x} (\sin 2x + \cos 2x).
\]

[36] \( y = \frac{\sec^7 x}{7} - \frac{\sec^5 x}{5} \). Using chain rule (or general power rule) we have
\[
\frac{dy}{dx} = \frac{1}{7} \sec^6 x \frac{d}{dx} [\sec x] - \frac{1}{5} \sec^4 x \frac{d}{dx} [\sec x] = \sec^6 x [\sec x \tan x] - \sec^4 x [\sec x \tan x] = \sec^7 x \tan x - \sec^5 x \tan x = (\sec^2 x - 1) \sec^5 x \tan x = \tan^2 x \sec^5 x \tan x \) (since \( \sec^2 x - \tan^2 x = 1 \)).
\]