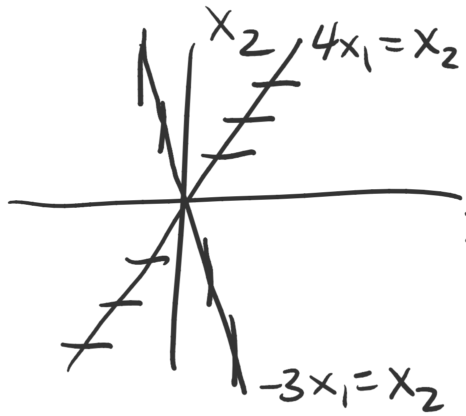
Discussion 5/14/2020

Thursday, May 14, 2020 5:03 PM

Plotting nullclines:

$\dot{x}_1 = 0 \rightarrow [X_2 + 3x_1 = 0 \Rightarrow x_2 = -3x_1$

$\dot{x}_2 = 0 \rightarrow [X_2 + {}^-4x_1 = 0 \Rightarrow x_2 = 4x_1$



$\dot{x}_1 = \frac{dx_1}{dt}$

$\dot{x}_2 = \frac{dx_2}{dt}$

Ch. 12 Probability

12.1 Principles of Counting

Fundamental Principle of Counting

- k selections to be made independently
- n_i possible choices, $i = 1, 2, \ldots, k$

Then the total number of choices: $n_1 \cdot n_2 \cdot \cdots \cdot n_k$

Ex How many possible CA license plates are there?

Rules:
- Total of 7 spots
- 1st spot is a 6
- Spots 2-4 are letters, I, O, Q cannot be in 2nd or 3rd spot

be in 2^{nd} or 3^{rd} spot

- Spots 5-7 are digits

e.g. $\underset{1}{\underline{6}}\ \underset{2}{\underline{A}}\ \underset{3}{\underline{B}}\ \underset{4}{\underline{C}}\ \underline{1}\ \underline{2}\ \underline{3}$

$n_1 = 1$ $\qquad$ $n_4 = 26$

$n_2 = 26 - 3 = 23 = n_3$ $\qquad$ $n_5 = n_6 = n_7 = 10$

$\#$ license plates $= 1 \cdot 23 \cdot 23 \cdot 26 \cdot 10 \cdot 10 \cdot 10$

$= 13{,}754{,}000$

Permutations

- set of ordered arrangements without repitition
- order matters

$\#$ of Permutations: $P_n = n(n-1)(n-2) \cdots 1 = n!$

Ex Suppose 10 horses are racing. How many outcomes are possible?

$$10 \cdot 9 \cdot \cdots 1 = 10!$$

$\#$ Permutations of n objects when k taken

$$P_{n,k} = \frac{n!}{(n-k)!}$$

Ex: Horse racing bets:

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Exacta : Horses place 1^{st} and 2^{nd} in exact order

Trifecta : Horses place 1^{st}, 2^{nd}, and 3^{rd} in order

Superfecta : Horses place 1^{st}, 2^{nd}, 3^{rd}, and 4^{th} in order

If 7 horses are racing, what are the total number of possible Exactas, Trifectas, and Superfectas?

$$\# \text{ Exactas: } P_{7,2} = \frac{7!}{(7-2)!} = \frac{7!}{5!} = 7 \cdot 6 = 42$$

$$\# \text{ Trifectas: } P_{7,3} = \frac{7!}{(7-3)!} = \frac{7!}{4!} = 7 \cdot 6 \cdot 5 = 210$$

$$\# \text{ Superfectas: } P_{7,4} = \frac{7!}{(7-4)!} = \frac{7!}{3!} = 7 \cdot 6 \cdot 5 \cdot 4 = 840$$

Combinations

- set of objects that is unordered, without repitition

\# of combinations of n objects when taking k

$$C_{n,k} = \frac{n!}{k!(n-k)!} = \binom{n}{k} \quad \text{"n choose k"}$$

Ex CA Mega Millions: \$266 Million estimated jackpot

How many possible lottery tickets?

Rules • Pick 5 numbers from 1 to 70, order does not matter

Rules
- Pick 5 numbers from 1 to 70, ... matter
- Pick 1 number from 1 to 25

$$\underbrace{\underline{7}\quad \underline{1}\quad \underline{6}\quad \underline{2}\quad \underline{3}}_{\text{1 to 70, no repeats}}\qquad \underbrace{\underline{14}}_{\text{1 to 25, separate pool}}$$

1 to 70

no repeats

1 to 25

separate pool

1^{st} 5 numbers : $C_{70,5} = \dfrac{70!}{5!(70-5)!}$

$$= \frac{70!}{5!\,65!}$$

$$= \frac{70\cdot 69\cdot 68\cdot 67\cdot 66\cdot \cancel{65!}}{5!\,\cancel{65!}}$$

$$= 12,103,014$$

$$\boxed{\begin{aligned} 70! &= 70\cdot 69! \\ &= 70\cdot 69\cdot 68! \\ &\vdots \end{aligned}}$$

Total available lotto tickets:

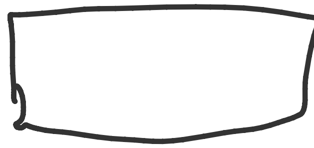
$$n_1\cdot n_2 = \underbrace{12,103,014}_{1^{st}\text{ 5 numbers}} \times \underbrace{25}_{\text{last number}} = 302,575,350$$

12.2 What is Probability

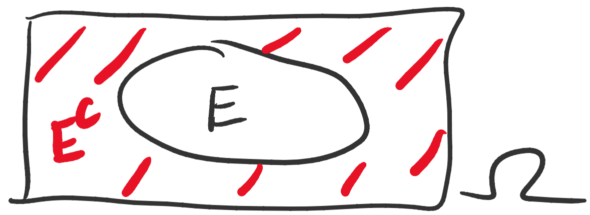
- Given an experiment with distinct $\underbrace{\text{possible outcomes}}$

- Given an experiment with distinct $\underbrace{\text{possible outcomes}}_{\text{Sample Space}}$

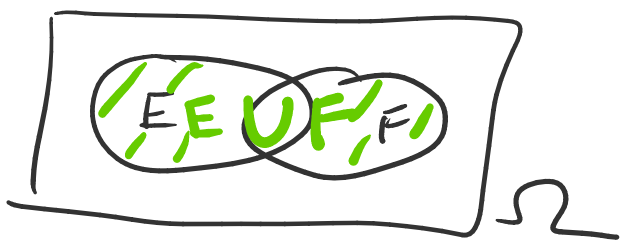
- Subset of sample space is called an event

Manipulating sets  Ω - sample space

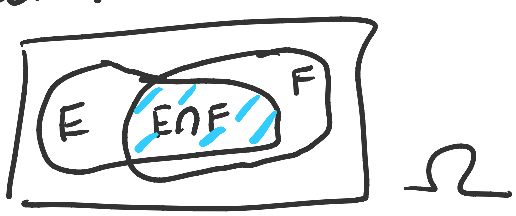
- complement of E, E^c



- union of E and F, $E \cup F$



- intersection of E and F, $E \cap F$



• E and F are disjoint if $E \cap F = \emptyset$

$E \cup F = F \cup E$

Commutative $E \cup F = F \cup E$

Associative $(E \cup F) \cup G = E \cup (F \cup G)$

Distributive $(E \cup F) \cap G = (E \cap G) \cup (F \cap G)$

Ex An experiment is tossing a coin and drawing a card from a deck.

a) How many elements are in the sample space?

b) How many elements are in the event "heads and an ace"

c) How many elements are in the event "tails and a face"

d) List the elements in "heads and spade"

H, T,  Draw 1 of 52 cards

a) # elements in sample space?

$2 \cdot 52 = 104$ elements in our sample space

b) "$\underbrace{\text{heads}}_{1}$ and an $\underbrace{\text{ace}}_{4 \text{ suits}}$"

1 4 suits

$$1 \cdot 4 = 4$$

c) "$\underbrace{\text{tails}}_{1}$ and a $\underbrace{\text{face}}$"

4 suits, 3 faces/suit] 12 faces

$$12 \cdot 1 = 12$$

d) "heads and spade"

$$\underbrace{H}_{1} \qquad \left.\begin{array}{c} A \\ 2 \\ \vdots \\ K \end{array}\right\} 13$$

13 elements, $\{\{H, A\}, \{H, 1\}, \ldots, \{H, K\}\}$