Discussion 5/14/2020

Thursday, May 14, 2020 4:07 PM

Ch. 12 Probability

12.1 Principles of Counting

Fundamental Principle of Counting

- K selections to be made independtly
- n_i possible choices, $i = 1, 2, \dots, K$

Then total number of choices: $n_1 \cdot n_2 \cdot \dots \cdot n_K$

<u>Ex</u> How many possible CA license plates are there?

<u>Rules</u>:
- Total of 7 spots
- 1^{st} spot is a 6
- Spots 2-4 are leters and Spot 2 and Spot 4 cannot be I, O, Q
- Spots 5-7 are digits

<u>e.g</u> <u>6</u> <u>A</u> <u>B</u> <u>C</u> <u>1</u> <u>2</u> <u>3</u>

$n_1 = 1$ $\qquad n_3 = 26$

$n_2 = 23 = n_4$ $\qquad n_5 = n_6 = n_7 = 10$

$\Rightarrow$ # license plates $= 1 \cdot 23 \cdot 26 \cdot 23 \cdot 10 \cdot 10 \cdot 10$

$\Rightarrow$ # license plates $= 1 \cdot 23 \cdot 26 \cdot 23 \cdot 10 \cdot 10 \cdot 10$
$= 13{,}754{,}000$

Permutations

- set of ordered arrangements without repitition

* order matters!

# of Permutations: $P_n = n(n-1)(n-2) \cdots 1 = n!$

Ex Suppose 10 horses are racing. How many outcomes are possible?

$10! \rightsquigarrow$ 1^{st} place : 10 choices
2^{nd} place : 9 choices
$\vdots$
10^{th} place : 1 choice

$\Rightarrow P_n = 10 \cdot 9 \cdot 8 \cdots 1 = 10!$

Number of permutations of n objects when K taken $P_{n,K} = \dfrac{n!}{(n-K)!}$

Ex Horse racing bets: Exacta: Horses place 1^{st} and 2^{nd} in order

Trifecta: Horses place 1^{st}, 2^{nd}, and 3^{rd} in

Trifecta: Horses place 1st, 2nd, and 3rd in exact order

Superfecta: Horses place 1st, 2nd, 3rd, and 4th in order.

If 7 horses are racing, what are the number of possible Exactas, Trifectas, and Superfectas?

\# Exactas: $\frac{7!}{(7-2)!} = \frac{7!}{5!} = 7 \cdot 6 = 42$

\# Trifectas: $\frac{7!}{(7-3)!} = \frac{7!}{4!} = 7 \cdot 6 \cdot 5 = 210$

\# Superfectas: $\frac{7!}{(7-4)!} = \frac{7!}{3!} = 7 \cdot 6 \cdot 5 \cdot 4 = 840$

Combinations

- set of objects that is unordered, without repitition

\# of combinations of n objects when taking k:

$$C_{n,k} = \frac{n!}{k!(n-k)!} = \binom{n}{k} \quad \text{"n choose k"}$$

Ex CA Mega Millions: $266 Million estimated jackpot

How many possible Jackpot options?

Rules: • 5 numbers from 1 to 70, order

Rules: • 5 numbers from 1 to 70, order does not matter
• 1 number from 1 to 25

1^{st} 5 numbers $C_{70,5} = \dfrac{70!}{5!\,(70-5)!}$

$$= \frac{70!}{5!\;65!}$$

$$= \frac{70 \cdot 69 \cdot 68 \cdot 67 \cdot 66}{5!}$$

$$= 12,103,014$$

Total number of choices when buying a lotto ticket is:

$$12,103,014 \times 25 = 302,575,350$$

12.2 What is Probability?

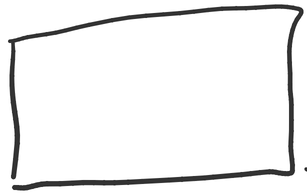
- Given an experiment with distinct $\underbrace{\text{possible outcomes}}_{\text{sample space}}$

- Subset of the sample space is called an event

- Subset of the samp…

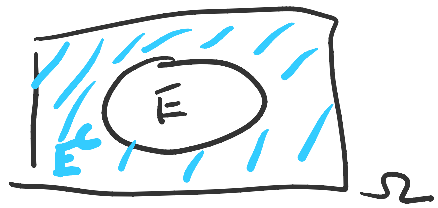
event

Manipulating sets

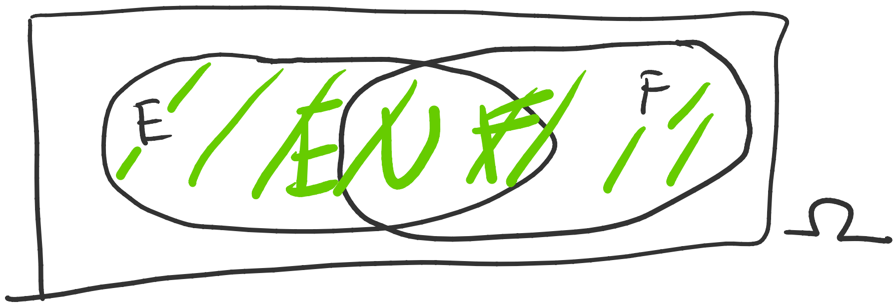


Ω - sample space

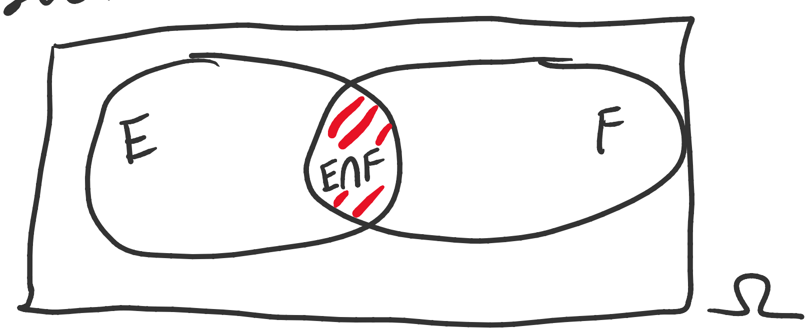
- complement of E, E^C



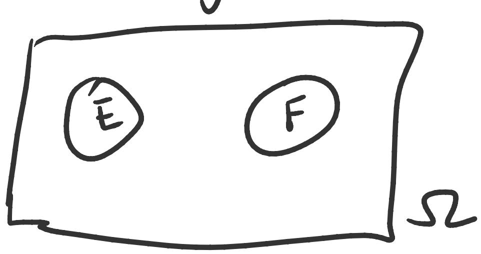
- union of E and F, $E \cup F$



- intersection of E and F, $E \cap F$



- E and F are disjoint : $E \cap F = \emptyset$



… $E \cup F = F \cup E$

Commutative: $E \cup F = F \cup E$

Associative: $(E \cup F) \cup G = E \cup (F \cup G)$

Distributive: $(E \cup F) \cap G = (E \cap G) \cup (F \cap G)$

Ex An experiment is tossing a coin and drawing a card from a deck.

a) How many elements are in the sample space?

b) How many elements in the event "heads and an ace"

c) How many elements in the event "tails and a face"

d) List the elements in "heads and spade"

→ 52 cards

2-sided coin

Flipping coin and draw 1 card

a) $2 \cdot 52 = 104$ elements in sample space

b) "$\underbrace{\text{heads}}_{1}$ and an $\underbrace{\text{ace}}_{4}$"

b) "$\underbrace{\text{heads}}_{1}$ and an $\underbrace{\text{ace}}_{4}$"

$$1 \cdot 4 = 4$$

c) "$\underbrace{\text{tails}}_{1}$ and a $\underbrace{\text{face}}$"

4 suits
3 faces per suit
12 face cards

$$12 \cdot 1 = 12$$

d) "heads and spade"

H ♠

$$\left.\begin{matrix} A \\ 2 \\ 3 \\ \vdots \\ J \\ Q \\ K \end{matrix}\right\} 13$$

H $\boxed{\text{Ace}}$

H $\boxed{2}$

$\vdots$

H $\boxed{K}$

$$1 \cdot 13 = 13$$