## Baye's Rule

$$P(E|F) = \frac{P(F|E)\,P(E)}{P(F)}$$

Ex Dengue-fever carried by mosquitoes.

- Of all mosquitoes that carry serotype A, 20% carry serotype B
- Frequency of serotype A is 10%
- Frequency of serotype B is 35%

What is the probability that a randomly selected mosquito carrying serotype B also carries serotype A?

$E = \{\text{mosquito carries serotype A}\}$

$F = \{\text{mosquito carries serotype B}\}$

Given $P(E) = 0.1$, $P(F) = 0.35$,
and $P(F|E) = 0.2$.

Then using Baye's rule

$$P(E|F) = \frac{P(F|E)\,P(E)}{P(F)}$$

$$= \frac{0.2 \cdot 0.1}{0.35}$$

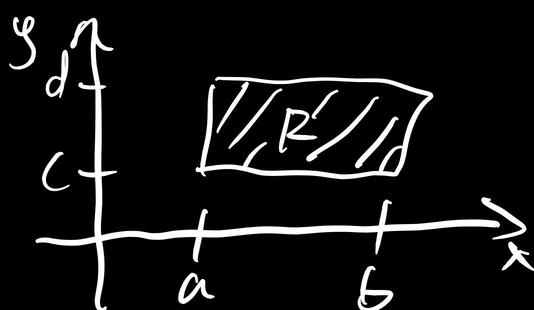
$$= \frac{2}{35} \approx 5.7\%$$

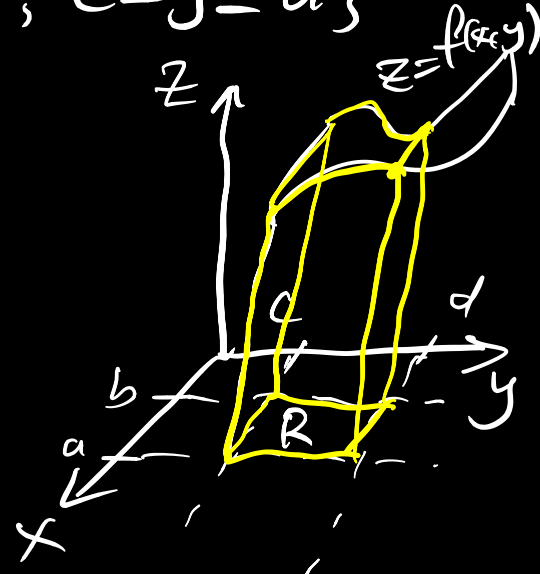
Then of all mosquitoes that carry serotype B, 5.7% also carry serotype A.

# Double Integrals

Integrating over rectangles

$$R = \{(x,y) \mid a \le x \le b,\ c \le y \le d\}$$




$$\text{Volume} = \iint_R f(x,y)\, dA$$

$$= \int_c^d \int_a^b f(x,y)\, dx\, dy$$

$$= \int_a^b \int_c^d f(x,y)\, dy\, dx$$

**Ex** Compute the double integral

$$\iint_R \frac{x}{1+xy}\,dA, \quad R=[0,1]\times[0,1]$$

$$\iint_R \frac{x}{1+xy}\,dA = \int_0^1\int_0^1 \frac{x}{1+xy}\,dy\,dx$$

$$= \int_0^1 \ln(1+xy)\Big|_0^1 \,dx$$

$\int \ln u = u\ln u - u + C$

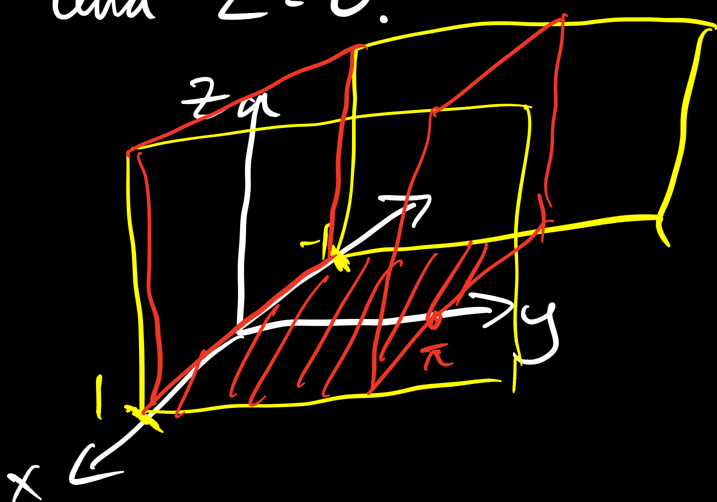
$$= \int_0^1 \ln(1+y) - \cancelto{0}{\ln(1)}\,dx$$

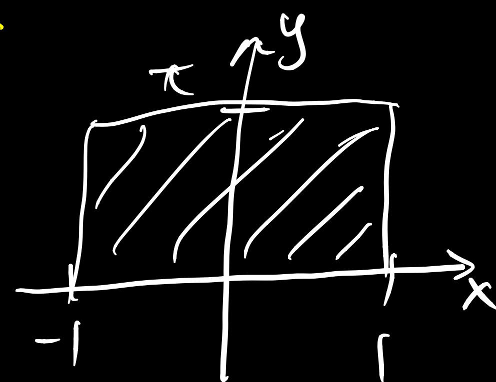
$$= (1+y)\ln(1+y) - (1+y)\Big|_0^1$$

$$= 2\ln(2) - 2 - (1\cdot\cancelto{0}{\ln(1)} - 1)$$

$$= 2\ln(2) - 1$$

Ex Find the volume of the solid enclosed by the surface $z = 1 + e^x \sin y$ and the planes $x = \pm 1$, $y = 0$, $y = \pi$, and $z = 0$.




$$\int_0^{\pi}\int_{-1}^{1} 1 + e^x \sin y \, dx\, dy$$

$$= \int_0^{\pi} x + e^x \sin y \Big|_{-1}^{1} dy$$

$$= \int_0^{\pi} 1 + e \sin y - (-1 + e^{-1} \sin y)\, dy$$

$$= \int_0^{\pi} 2 + (e - e^{-1}) \sin y \, dy$$

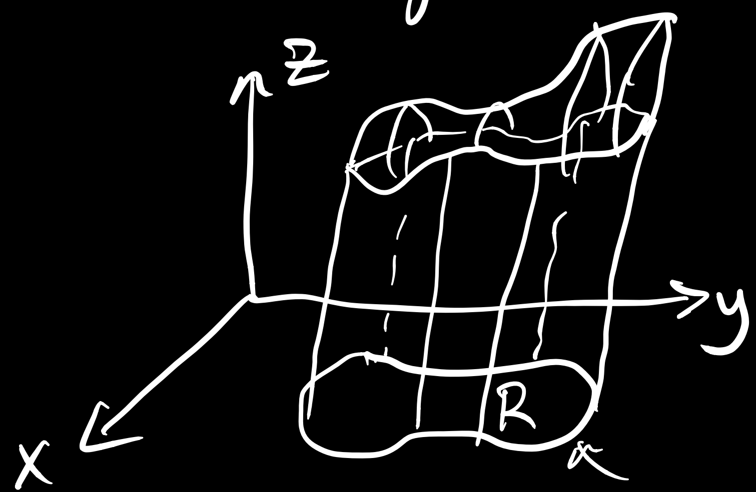
$$= 2y + (e - e^{-1})(-\cos y) \Big|_0^{\pi}$$

$$= 2\pi + (e - e^{-1})(+1)$$

$$- 0 - (e - e^{-1})(-1)$$

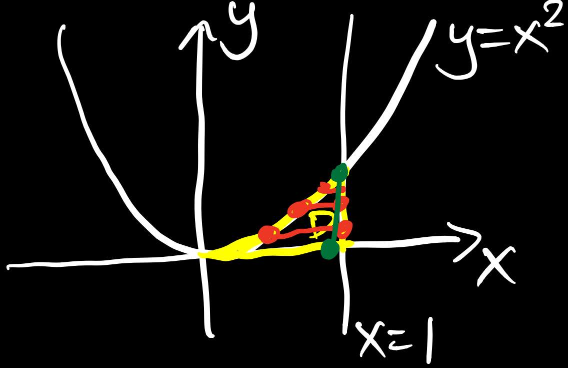
$$= 2\pi + 2(e - e^{-1})$$

Double Integrals over general regions




Ex Evaluate the double integral

$\iint_D x\cos y \, dA$, D bounded by $y=0$, $y=x^2$, and $x=1$.




$\rightarrow x = \pm\sqrt{y}$

$$\iint_D x\cos y \, dA = \int_0^1 \int_{\sqrt{y}}^1 x\cos y \, dx \, dy$$

$$= \int_0^1 \frac{x^2}{2}\cos y \Big|_{\sqrt{y}}^1 dy$$

$$= \int_0^1 \frac{1}{2}\cos y + \frac{-y}{2}\cos y \; dy$$

$u = \frac{-y}{2}$ $\quad dv = \cos y \, dy$

$du = \frac{-dy}{2}$ $\quad V = \sin y$

$$= \int_0^1 \frac{1}{2}\cos y \, dy + \frac{-y}{2}\sin y \Big|_0^1$$

$$- \int_0^1 -\frac{1}{2}\sin y \, dy$$

$$= \frac{1}{2}\sin y \Big|_0^1 - \frac{y}{2}\sin y \Big|_0^1 - \frac{1}{2}\cos y \Big|_0^1$$

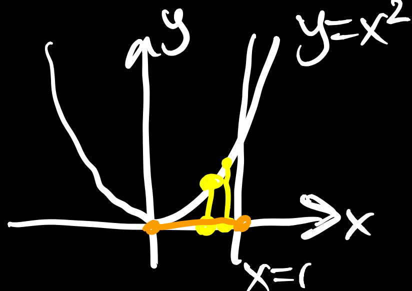
$$= \frac{1}{2}\sin(1) - \frac{1}{2}\sin(1) - \frac{1}{2}\left(\cos(1) - \cos(0)\right)$$

$$= -\frac{1}{2}\left(\cos(1) - 1\right)$$

$$= \frac{1}{2}\left(1 - \cos(1)\right)$$

We integrated with respect to x then y. Can we instead integrate with respect to y then x?

Yes! But we have to change the integrating bounds

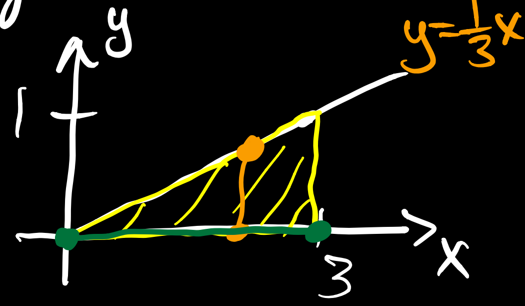



$$\iint_D x\cos y \, dA$$

$$= \int_0^1 \int_0^{x^2} x\cos y \, dy\, dx$$

Ex Evaluate the integral by reversing the order of integration.

$$\int_0^1 \int_{3y}^3 e^{x^2}\,dx\,dy$$




$x = 3y$

$y = \frac{1}{3}x$

$$\int_0^1 \int_{3y}^3 e^{x^2}\,dx\,dy = \int_0^3 \int_0^{\frac{1}{3}x} e^{x^2}\,dy\,dx$$

$$= \int_0^3 y e^{x^2} \Big|_0^{\frac{1}{3}x} dx$$

$$= \int_0^3 \frac{1}{3} x e^{x^2}\,dx$$

$$= \frac{1}{6} e^{x^2} \Big|_0^3$$

$$= \frac{1}{6}\left(e^9 - 1\right)$$

# Finding the Area of a Region

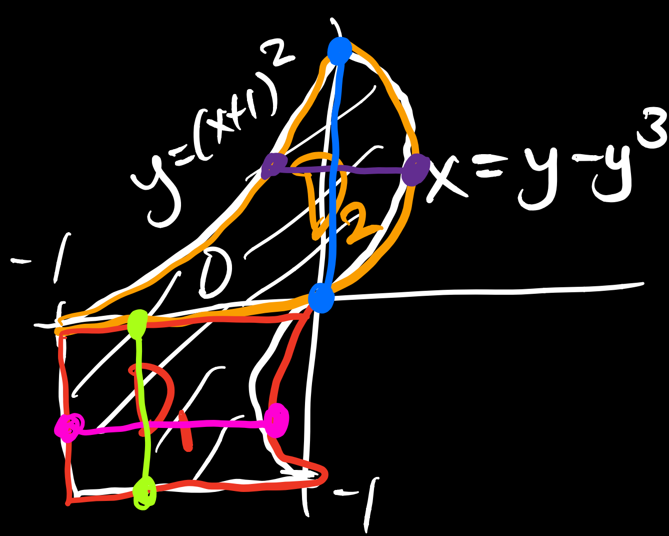
→ integrate $f(x,y)=1$ over the region


$V = l \cdot w \cdot h$

$A = l \cdot w$

→ if $h=1$, $V=A$

**Ex** Find the area of the region $D$




$$\text{Area} = \iint_D 1\, dA = \iint_{D_1} dA + \iint_{D_2} dA$$

$$= \int_{-1}^{0} \int_{-1}^{y-y^3} dx\,dy + \int_{0}^{1} \int_{\sqrt{y}-1}^{y-y^3} dx\,dy$$