Baye's Rule

$$P(E|F) = \frac{P(F|E)\,P(E)}{P(F)}$$

Ex Dengue-fever carried mosquitoes

- Of all mosquitoes that carry serotype A, 20% carried serotype B
- Frequency of serotype A is 10%
- Frequency of Serotype B is 35%

What is the probability that a randomly selected mosquito carrying serotype B also carries serotype A?

$$E = \{\text{mosquito has serotype } A\}$$

$F = \{\text{mosquito has serotype B}\}$

Given: $P(E) = 0.1$, $P(F) = 0.35$,
and $P(F|E) = 0.2$

Then using Baye's rule

$$P(E|F) = \frac{P(F|E)P(E)}{P(F)}$$

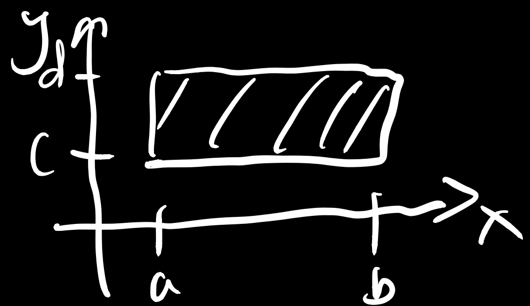
$$= \frac{0.2 \cdot 0.1}{0.35}$$

$$= \frac{2}{35} \approx 5.7\%$$

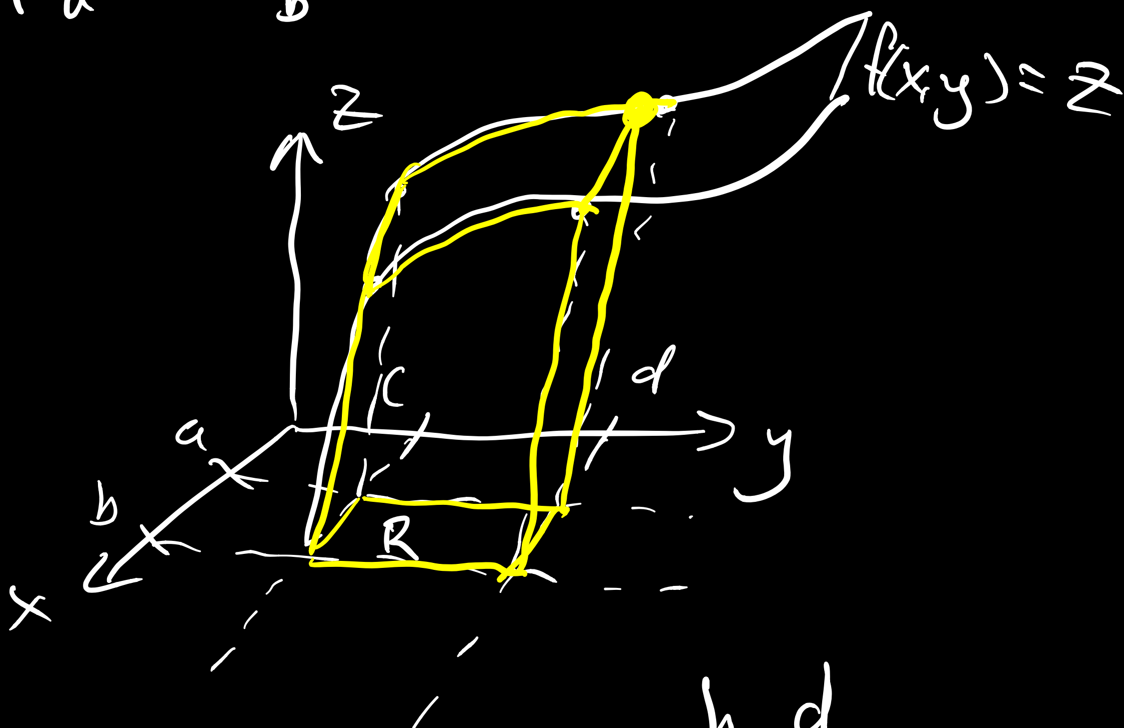
Then, of all mosquitoes that carry serotype B, 5.7% also carry serotype A.

Double Integrals

Integrating over rectangles



$$R = \{(x,y) \mid a \le x \le b,\ c \le y \le d\}$$



$$\text{Volume} = \iint_R f(x,y)\,dA = \int_a^b \int_c^d f(x,y)\,dy\,dx$$

Ex Compute the double integral 

$$\iint_R \frac{x}{1+xy}\,dA, \quad R=[0,1]\times[0,1].$$

Recall: $\int \ln u\,du = u\ln u - u + C$

$$\iint_R \frac{x}{1+xy}\,dA = \int_0^1\int_0^1 \frac{x}{1+xy}\,dy\,dx$$

$$= \int_0^1 \ln(1+xy)\Big|_0^1\,dx$$

$$= \int_0^1 \ln(1+x) - \cancel{\ln(1)}^{\,0}\,dx$$

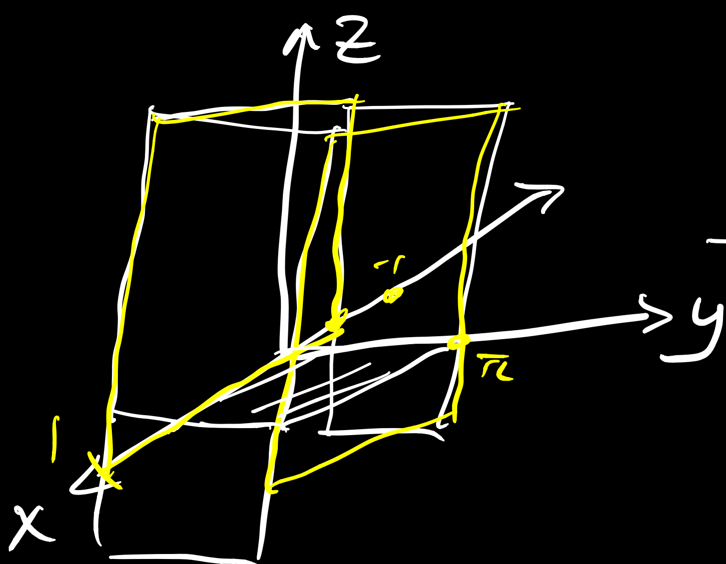
$$= (1+x)\ln(1+x) - (1+x)\Big|_0^1$$

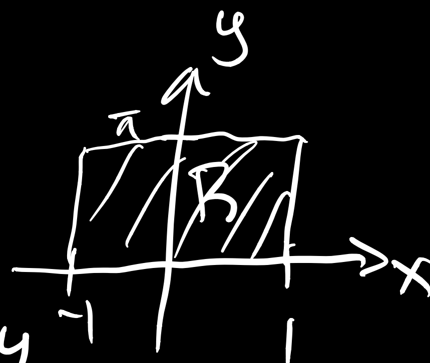
$$= 2\ln(2) - 2 - \left[1\cdot\cancel{\ln(1)}^{\,0} - 1\right]$$

$$= 2\ln(2) - 1$$

$$\frac{\partial}{\partial y}\ln(1+xy) = \frac{x}{1+xy}$$

Ex Find the volume of the solid enclosed by the surface $z = 1 + e^x \sin y$ and the planes $x = \pm 1$, $y = 0$, $y = \pi$, and $z = 0$.





$$\iint_R 1 + e^x \sin y \, dA = \int_0^\pi \int_{-1}^1 1 + e^x \sin y \, dx\, dy$$

$$= \int_0^\pi x + e^x \sin y \Big|_{-1}^1 \, dy$$

$$= \int_0^\pi 1 + e\sin y - (-1 + e^{-1}\sin y) \, dy$$

$$= \int_0^{\pi} 2 + (e - e^{-1}) \sin y \, dy$$

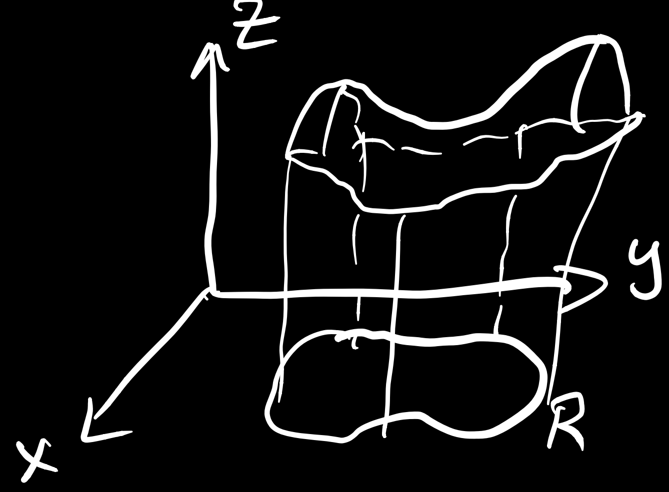
$$= 2y + (e - e^{-1})(-\cos y) \Big|_0^{\pi}$$

$$= 2\pi + (e - e^{-1})(1)$$

$$- (e - e^{-1})(-1)$$

$$= 2\pi + 2(e - e^{-1})$$

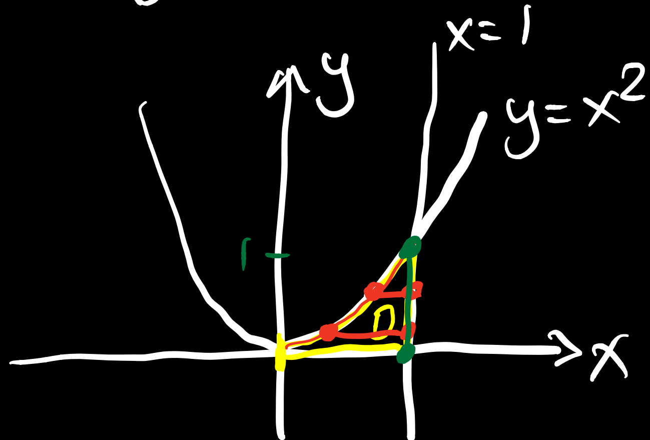
Double Integrals over general regions



<u>Ex</u> Evaluate the double integral

$\iint_D x\cos y\, dA$, D bounded by

$y=0,\ y=x^2,\ x=1.$



$\longrightarrow x = \pm\sqrt{y}$

$$\iint_D x\cos y\, dA = \int_0^1 \int_{\sqrt{y}}^1 x\cos y \; dx\, dy$$

$$= \int_0^1 \frac{x^2}{2}\cos y \Big|_{\sqrt{y}}^1 dy$$

$$= \int_0^1 \frac{1}{2}\cos y - \frac{y}{2}\cos y \; dy$$

$u = \frac{y}{2}$ $\quad dv = \cos y\, dy$

$du = \frac{dy}{2}$ $\quad v = \sin y$

$$= \int_0^1 \frac{1}{2}\cos y - \frac{y}{2}\sin y \Big|_0^1$$

$$+ \int_0^1 \frac{1}{2}\sin y \, dy$$

$$= \frac{1}{2}\sin y \Big|_0^1 - \frac{y}{2}\sin y \Big|_0^1$$

$$+ -\frac{1}{2}\cos y \Big|_0^1$$

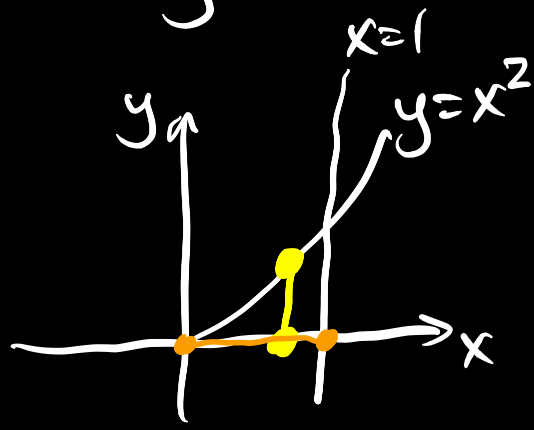
$$= \frac{1}{2}\sin(1) - \frac{1}{2}\sin(1) + -\frac{1}{2}(\cos(1) - \cos(0))$$

$$= -\frac{1}{2}(\cos(1) - 1)$$

$$= \frac{1}{2}(1 - \cos(1))$$

We integrated with respect to x then y. Can we integrate with respect to y then x? Yes!

$$\int_0^1 \int_{\sqrt{y}}^1 x\cos y \, dx\,dy = \int_0^1 \int_0^{x^2} x\cos y \, dy\,dx$$



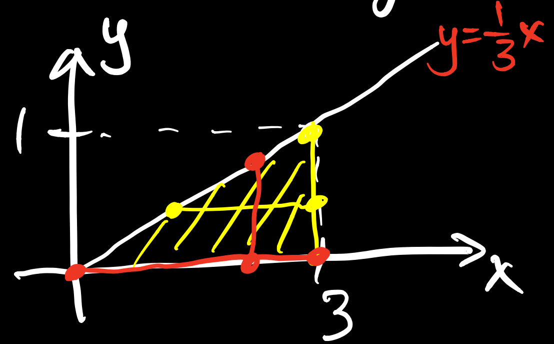
$$= \frac{1}{2}(1-\cos(1))$$

Ex Evaluate the integral by reversing the order of integration.

$$\int_0^1 \int_{3y}^3 e^{x^2} \, dx\,dy$$

$x = 3y$

$y = \frac{1}{3}x$



$$\int_0^3 \int_0^{\frac{1}{3}x} e^{x^2} \, dy\,dx = \int_0^3 y e^{x^2} \Big|_0^{\frac{1}{3}x} dx$$

$$= \int_0^3 \frac{1}{3} x e^{x^2} \, dx$$

$$= \frac{1}{6} e^{x^2} \Big|_0^3$$

$$= \frac{1}{6}(e^9 - 1)$$

Finding Area of a Region

→ integrate $f(x,y) = 1$ over the region

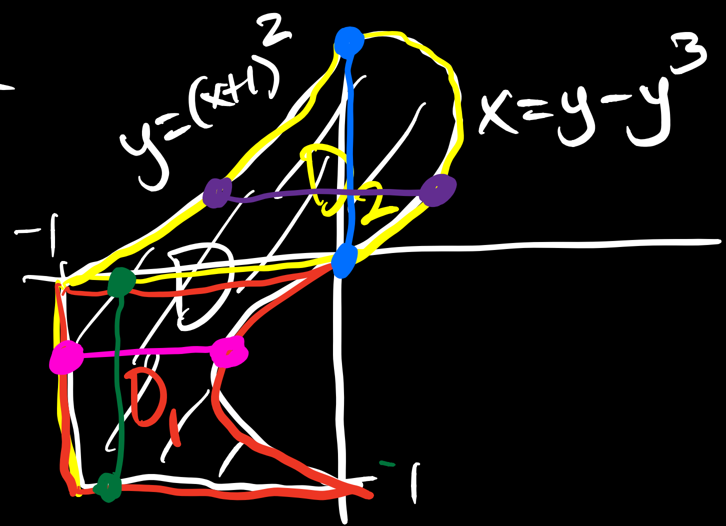
$V = l \cdot w \cdot h$

Area of base $= l \cdot w$

If $h = 1$ then

$V =$ Area of base

Ex



$$\iint_D 1\,dA = \iint_{D_1} dA + \iint_{D_2} dA$$

$$= \int_{-1}^{0}\int_{-1}^{y-y^3} dx\,dy + \int_{0}^{1}\int_{\sqrt{y}-1}^{y-y^3} dx\,dy$$