Qualitative Analyzers of Linear Systems

2D system of differential equations

\[
\frac{d\bar{x}}{dt} = A(t)\bar{x} + \bar{g}(t), \quad \bar{x} = \begin{bmatrix} x \\ y \end{bmatrix}
\]

\( A \) is a matrix
\( \bar{g} \) is a vector
\( \bar{x} \) is a vector

Classifications

1. If \( A \) and \( g \) are independent of \( t \), then the system is called autonomous
   \[ \rightarrow \text{Otherwise, nonautonomous} \]

2. If \( \bar{g} = 0 \), then the system is called homogeneous
   \[ \rightarrow \text{Otherwise, nonhomogeneous} \]

Qualitative Features

- **Equilibria** - points where \( x'(t) = 0 \) and \( y'(t) = 0 \)

- **Nullclines** - curves where \( x'(t) = 0 \) or \( y'(t) = 0 \)

- **Stability** - classification of equilibria
  - **Stable** - if you start near the equilibrium, you end up there as \( t \to \infty \)
  - **Unstable** - not stable

- **Types of equilibria** - saddles, nodes, spirals
\[ \text{Saddle} \]

\[ \text{Node} \quad (\text{Stable node}) \]

\[ \begin{aligned} y' &= 0 \\ x' &= 0 \end{aligned} \]

- Unstable node will have arrows reversed.

\[ \text{Spiral} \quad (\text{Stable spiral}) \]

\[ \begin{aligned} y' &= 0 \\ x' &= 0 \end{aligned} \]

- Unstable spiral will have arrows reversed.