Ben Godkin
MSB 2125
bmgodkin@ucdavis.edu
OH: MW 5-6 PM Academic Surge 2142
Calc. Room: MW 4-5 PM

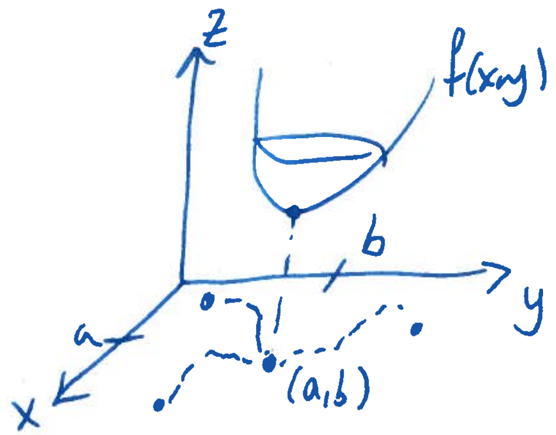
Last time: 1. Introduction to functions of several variables

Examples: Topographic maps, BMI, Wind-chill

2. Level curves: $f(x,y) = K$, K a constant.

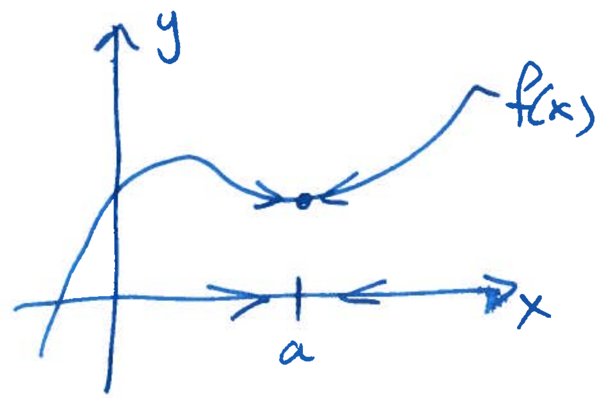
Limits and Continuity

$$\lim_{(x,y)\to(a,b)} f(x,y) = L$$



1D

$$\lim_{x\to a} f(x) = L$$

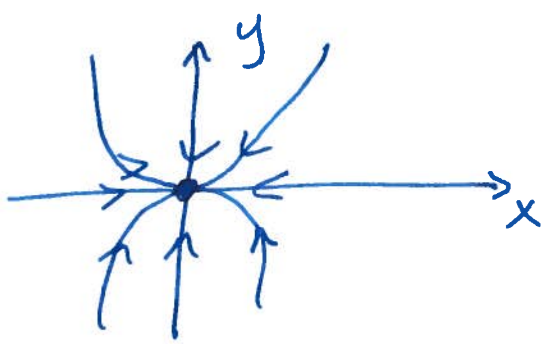


Non-existence of limits

If $f(x,y) \to L_1$ along C_1 and $f(x,y) \to L_2$ along C_2, and $L_1 \neq L_2 \Rightarrow \lim_{(x,y)\to(a,b)} f(x,y)$ does not exist!

Ex Determine whether the limit $\lim_{(x,y)\to(0,0)} f(x,y)$ exists for $f(x,y) = \dfrac{x^2+\sin^2 y}{2x^2+y^2}$

~~Domain~~



Paths

$x=0$

$y=0$

$y=x$

$y=-x$

$y=x^2$

$\vdots$

~~$C_1 : (0,y)$~~

~~Ex~~

$C_1 : x=0$. Points on C_1 look like $(0,y)$. Then along C_1,

$$f(x,y) = f(0,y) = \frac{\sin^2 y}{y^2}$$

Now, we compute

$$\lim_{y\to 0} f(0,y) = \lim_{y\to 0} \frac{\sin^2 y}{y^2} \quad = \text{"}\frac{0}{0}\text{"}$$

$$= \lim_{y\to 0} \frac{2\sin y\cos y}{2y} \quad (\text{l'Hopital}) \quad \Leftarrow \text{"}\frac{0}{0}\text{"}$$

$$= \lim_{y\to 0} \frac{\cos^2 y - \sin^2 y}{1}$$

$$= \frac{1-0}{1}$$

$$= 1.$$

$\Rightarrow$ Along C_1, $\lim_{(x,y)\to(0,0)} f(x,y) = 1.$

C_2: $y = 0$. Points on C_2 look like $(x,0)$. Then along C_2,

$$f(x,y) = f(x,0) = \frac{x^2}{2x^2} = \frac{1}{2}$$

$$\Rightarrow \lim_{x\to 0} f(x,0) = \frac{1}{2}$$

$\Rightarrow$ Along C_2, $\lim_{(x,y)\to(0,0)} f(x,y) = \frac{1}{2}$.

$1 \neq \frac{1}{2}$, so we found two different limits along two different curves, so $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist!

Continuity

f is continuous at (a,b) if

$$\lim_{(x,y)\to(0,0)} f(x,y) = f(a,b)$$

i.e. "The limit and function value agree"

Partial Derivatives

→ Same as before, but treat other variables as constants!

Clairaut's Theorem

If f_{xy} and f_{yx} are continuous, then

$$f_{xy}(a,b) = f_{yx}(a,b)$$

for any point (a,b).

Ex Find all the third order partial derivatives of

$$f(x,y) = \sqrt{x^2+y^2}$$

$$f_x = \frac{1}{2\sqrt{x^2+y^2}} 2x$$

$$= \frac{x}{\sqrt{x^2+y^2}}$$

$$= \frac{x}{f(x,y)}$$

$$f_{xx} = \frac{f(x,y) \cdot 1 - x f_x}{f^2}$$

$$= \frac{1}{f} - \frac{x}{f^2} \cdot \frac{x}{f}$$

$$= \frac{1}{f} - \frac{x^2}{f^3}$$

$$= (f)^{-1} - x^2 (f)^{-1}$$

$$f_{xxx} = -(f)^{-2} f_x - \left(2x(f)^{-1} + x^2(-1)(f)^{-1} f_x\right)$$

$$= -(f)^{-2} \frac{x}{f} - \frac{2x}{f} + \frac{x^2}{f} \cdot \frac{x}{f}$$

$$= -\frac{x}{f^3} - \frac{2x}{f} + \frac{x^3}{f^2}$$