Conditional Probability

Consider events $E$ and $F$ with $P(F)$. The probability of $E$ given $F$ is

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

"We know $F$ happened so it is now our universe."

"Only way for $E$ to happen is when $E \cap F$"

Complement rule for conditional probability

$$P(E^c|F) = 1 - P(E|F)$$
Multiplication Rule and Independence

Multiplication rule: \( P(ENF) = P(E|F)P(F) \)

Independence

\( E \) and \( F \) independent if \( P(ENF) = P(E)P(F) \)

Law of Total Probability

\( F_1, F_2, \ldots, F_k \) pairwise disjoint, exhaustive events. Then

\[
P(E) = P(E|F_1)P(F_1) + \cdots + P(E|F_k)P(F_k)
\]

Bayes' Rule

\[
P(F|E) = \frac{P(E|F)P(F)}{P(E)}
\]
Discrete Random Variables

- a way to assign events to numbers
- $X$ takes in an element of sample space and outputs a number
- "$X=i$" means a particular element of sample space taking place
- $P(X=i)$ is the probability of that outcome

Probability density function (PDF)

$P_i = P(X=i)$

- $P_i \geq 0$, $\forall i$
- $\sum_i P_i = 1$

Cumulative distribution function (CDF)

$F_i = P(X \leq i)$

"What is the probability of events up to $i$ occurring?"
**Mean and Variance of Discrete Random Variables**

*Expected value/mean (center of mass)*:

\[ E[X] = \sum_i i p_i \]

"What are your expected winnings?"

*Variance*:

\[ \text{Var}[X] = \sum_i (i - E[X])^2 p_i \]

"How far do you go away from the mean?"

*Standard deviation*:

\[ \sigma = \sqrt{\text{Var}[X]} \]

**Bernoulli Random Variables**

- Takes values in \( \{0, 1\} \)
  
  * \( P(X=1) = p \)
  
  \[ P(X=0) = 1 - p \]