MAT 22A Midterm Topics

Below is a list of topics that will be covered on the final exam. The list shows the major topics that we have covered in lecture, but it may not be an exhaustive list.

1. Introduction to Vectors
   - Vectors, properties of vectors, and vector algebra (addition, scalar multiplication, dot product, lengths, inequalities)
   - Linear combinations of vectors
   - Geometric picture of vectors and linear combinations
   - Linear independence of vectors
   - Matrices and matrix algebra (addition, scalar multiplication, matrix-matrix multiplication, inverses, special types of matrices, block matrices)
   - Perspectives of matrix-vector/matrix multiplication
2. Solving Linear Systems
   - Linear equations, matrix representation, and geometric interpretations
   - Elimination using elimination/permutation matrices or the augmented matrix
   - Finding inverses of matrices using Gauss-Jordan elimination or other means
   - Factorizations of A: $A = LU$, $PA = LU$, $A = LDU$
   - Transposes of matrices
3. Vector Space and Subspaces
   - Vector Spaces: definition and examples
   - Subspaces: definition and examples
   - Column Space of A
   - Span of a collection of vectors
   - Nullspace of A
   - Reduced row echelon form (rref) of A
   - Rank of a matrix
   - The complete solution
   - Full column rank, full row rank, and other situations
   - Finding bases of vectors spapces and determining the dimension of a vector space
   - Four fundamental subspaces
   - Rank-nullity theorem
   - Finding bases of the each of the four fundamental subspaces
   - Determining the dimension of each of the four fundamental subspaces
4. Orthogonality
   - Orthogonality of the four fundamental subspaces (which spaces are orthogonal)
   - Orthogonal subspaces and orthogonal complements
   - Projections, projection matrices, orthogonality of the error vector
   - Derive/reason how to determine a projection matrix
   - Derive/reason what is and how to find a lease squares solution
   - Properties of projection matrices
   - Determine a least squares solution
   - Polynomial interpolation using least squares

- Orthogonal matrices and their properties
- Matrices whose columns are orthonormal
- Projections and least squares using matrices with orthonormal columns
- Using Gram-Schmidt to find an orthogonal basis
- A=QR factorization

5. Determinants
    - Information "stored" in the determinant
    - Compute determinants of matrices
    - Properties of determinants
6. Eigenvalues and Eigenvectors
    - Finding eigenvalues and eigenvectors
    - Relationship between determinant and trance and eigenvalues
    - Geometric interpretation of eigenvalues and eigenvectors
    - Geometric and arithmetic multiplicity
    - Diagonalization of a matrix
    - Symmetic matrices
    - Definition of a positive definite matrix
    - Determining whether a matrix is positive definite

Some advice:

- Review your notes and make sure you understand the topics and examples we have covered during lecture.
- Review the problem sets and make sure you understand how to do the problems without the solution keys.
- Practice the problems from the problem sets and make new problems by changing the numbers and/or size of the matrices in the problem sets. You can check your answers using MATLAB.
- Stop by office hours, and we can chat about any material you are struggling with, extra examples, or anything else!
- Attend the review session on Thursday 7/30 from 3PM-5PM.