MAT 22A Midterm (Summer Session I 2020)

1. Determine whether the following statements are true or false. (No work required) (18 points)

   (a) The linear combination $a\begin{bmatrix}1\\2\\1\end{bmatrix}+b\begin{bmatrix}1\\0\\1\end{bmatrix}$ fills a line. a and b are real numbers.

   (b) The vectors $\begin{bmatrix}1\\0\\-1\end{bmatrix}$ and $\begin{bmatrix}2\\1\\1\end{bmatrix}$ are orthogonal.

   (c) $\mathbf{x}=\mathbf{0}$ is a solution to $A\mathbf{x}=\mathbf{0}$.

   (d) If A is an $k\times n$ matrix and B is an $n\times \ell$ matrix, then $(AB)^{\mathrm{T}}$ is an $\ell\times k$ matrix.

   (e) If A and B are symmetric matrices, then $AB=BA$.

   (f) If the columns of A are linearly dependent, then A is invertible.

   (g) $\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}\begin{bmatrix}— & \mathbf{a}_1 & —\\— & \mathbf{a}_2 & —\\— & \mathbf{a}_3 & —\end{bmatrix}$ produces the matrix $\begin{bmatrix}— & \mathbf{a}_1 & —\\— & \mathbf{a}_3 & —\\— & \mathbf{a}_2 & —\end{bmatrix}$.

   (h) The set of $n\times n$ upper triangular matrices are a subspace of the vector space of $n\times n$ matrices.

   (i) If A is an $m\times n$ matrix and $m<n$, then $A\mathbf{x}=\mathbf{0}$ has a nonzero solution.

2. Sketch the column picture and row picture of the following system. (7 points)

$$\begin{aligned}2x+y&=-1\\-x+2y&=1\end{aligned}$$

3. Solve the following system using the inverse of the coefficient matrix. (10 points)

$$\begin{aligned}2x_1-4x_2-6x_3&=0\\x_1+2x_2+1x_3&=4\\-2x_1+4x_3&=-1\end{aligned}$$

4. Find the LU-decomposition of the coefficient matrix and use back-substitution followed by forward-substitution to solve the following system. (15 points)

$$\begin{aligned}4x_1+2x_2+-2x_3&=4\\2x_1-x_2-x_3&=6\\3x_1+x_3&=-7\end{aligned}$$

5. Find the nullspace of the following matrix and determine the rank of A. (10 points)

$$A=\begin{bmatrix}1&0&2&1&1\\2&1&-1&0&1\\1&-2&0&2&-1\end{bmatrix}$$