1. Determine whether the following statements are true or false. (No work required) (18 points)

(a) The linear combination \(a \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + b \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\) fills a line. \(a\) and \(b\) are real numbers.

(b) The vectors \(\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}\) and \(\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}\) are orthogonal.

(c) \(x = 0\) is a solution to \(Ax = 0\).

(d) If \(A\) is an \(k \times n\) matrix and \(B\) is an \(n \times \ell\) matrix, then \((AB)^T\) is an \(\ell \times k\) matrix.

(e) If \(A\) and \(B\) are symmetric matrices, then \(AB = BA\).

(f) If the columns of \(A\) are linearly dependent, then \(A\) is invertible.

(g) \(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\) produces the matrix \(\begin{bmatrix} -a_1 \\ -a_2 \\ -a_3 \end{bmatrix}\).

(h) The set of \(n \times n\) upper triangular matrices are a subspace of the vector space of \(n \times n\) matrices.

(i) If \(A\) is an \(m \times n\) matrix and \(m < n\), then \(Ax = 0\) has a nonzero solution.

2. Sketch the column picture and row picture of the following system. (7 points)

\[
\begin{align*}
2x + y &= -1 \\
-x + 2y &= 1
\end{align*}
\]

3. Solve the following system using the inverse of the coefficient matrix. (10 points)

\[
\begin{align*}
2x_1 - 4x_2 - 6x_3 &= 0 \\
x_1 + 2x_2 + 1x_3 &= 4 \\
-2x_1 + 4x_3 &= -1
\end{align*}
\]

4. Find the \(LU\)-decomposition of the coefficient matrix and use back-substitution followed by forward-substitution to solve the following system. (15 points)

\[
\begin{align*}
4x_1 + 2x_2 + -2x_3 &= 4 \\
2x_1 - x_2 - x_3 &= 6 \\
3x_1 + x_3 &= -7
\end{align*}
\]

5. Find the nullspace of the following matrix and determine the rank of \(A\). (10 points)

\[
A = \begin{bmatrix}
1 & 0 & 2 & 1 & 1 \\
2 & 1 & -1 & 0 & 1 \\
1 & -2 & 0 & 2 & -1
\end{bmatrix}
\]