MAT 22A Extra Problem Set

1. Let S be the set of $n \times n$ matrices that are similar to some $n \times n$ matrix A. Is S a subspace of $\mathbb{R}^{n \times n}$?

2. Suppose A and B are $n \times n$ matrices that have the same set of eigenvectors. Show that $AB = BA$.

3. Prove that if A is a real $n \times n$ matrix, then any complex eigenvalues come in conjugate pairs. Argue that if n is odd, then A must have a real eigenvalue.

4. Find an orthogonal matrix that diagonalizes

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & -2 \\ 2 & -2 & 0 \end{bmatrix}.$$

5. For what values of a and b are the following matrix positive definite

   (a) $A = \begin{bmatrix} 1 & b \\ b & 9 \end{bmatrix}$

   (b) $B = \begin{bmatrix} 2 & 4 \\ 4 & c \end{bmatrix}$

   (c) $C = \begin{bmatrix} c & b \\ b & c \end{bmatrix}$