MAT 22A Problem Set 10 (Due 7/27 11:59 PM)

1. Consider the plane $x + y + z = 0$ as a subspace of $\mathbb{R}^3$. Choose two orthogonal vectors and make them orthonormal. Finally, find the projection matrix and projection of $\mathbf{b} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$ on the plane. Make a rough sketch of the plane, $\mathbf{b}$, and the projection of $\mathbf{b}$ onto the plane.

2. Find an orthonormal basis of the following vector space S.

$$S = \text{span}\left(\begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 7 \\ 2 \\ 3 \end{bmatrix} \right)$$

3. Suppose the columns of an $m \times n$ matrix A are orthogonal. Consider the system $A\mathbf{x} = \mathbf{b}$. Find an expression for x_i (the i^{th} entry of $\mathbf{x}$).

4. Consider the following system $A\mathbf{x} = \mathbf{b}$. Find the $A = QR$ decomposition of the following matrix. Then solve the system using the decomposition.

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$