

1. Compute the determinant of A .

$$A = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 2 & 1 & 2 & 1 \\ 1 & 0 & -1 & 0 \\ -2 & 1 & 0 & 0 \end{bmatrix}.$$

2. Find the eigenvalues and eigenvectors of A .

$$A = \begin{bmatrix} -3 & -4 & -5 & -1 \\ 4 & 4 & 4 & 4 \\ -4 & -2 & -2 & -4 \\ 3 & 6 & 3 & 1 \end{bmatrix}.$$

3. Find the $A = X\Lambda X^{-1}$ decomposition of A from problem 2. X is a matrix whose columns are the eigenvectors of A and Λ is a diagonal matrix whose entries are the eigenvalues of A . Next, compute the matrix products to show that indeed $A = X\Lambda X^{-1}$. Determine the eigenvalues of A^k and diagonalize A^k .