MAT 22A Problem Set 3 (Due 7/1, 8 AM)

1. Find the multipliers ℓ_{ij} to reduce the following systems to an upper triangular system:

   (a)

$$\begin{aligned} 2x + 3y + z &= 1 \\ 2x + 3y - z &= 2 \\ 3x + y + 2z &= 1 \end{aligned}$$

   (b)

$$\begin{aligned} ax + by &= c \\ dx + ey &= f \end{aligned}$$

2. Solve the following system using elimination to reduce the system to a triangular system. Then use back substitution to find the solution.

$$\begin{aligned} x + y + z &= -1 \\ 2x - y - 3z - t &= 1 \\ x + y + t &= 2 \\ 4x - z + 2t &= -1 \end{aligned}$$

3. Find c so that the linear system below

   (a) requires a row exchange.
   (b) is singular.
   (c) does not require a row exchange.

$$\begin{aligned} x + 2y + z &= 4 \\ 3x + cy + 3z &= 2 \\ -y + z &= 1 \end{aligned}$$

   Finally, find the solution to the system in terms of c.

4. Let $A \in \mathbb{R}^{n\times n}$. Use the matrix-matrix multiplication definition to show that $AI = A$ where I is the identity matrix.

5. Let $A \in \mathbb{R}^{m\times n}$ and $B, C \in \mathbb{R}^{n\times \ell}$. Use the matrix-matrix multiplication definition to show that $A(B + C) = AB + AC$.

6. Find an example of A and B (4×4 matrices) such that

   (a) $AB = BA$.
   (b) $AB \neq BA$.

Assume A and B are not the zero matrix or the identity matrix.

7. Consider the following system of linear equations

$$\begin{aligned} 2y - z + t &= 1 \\ 3x + 4y - z &= 1 \\ x - 2y + t &= 1 \\ x + t &= 1. \end{aligned}$$

   Solve the system using a squence of elimination matrices E_{ij} and permutation matrices P_{ij}. Let C be the product of the matrices used to reduce the system to triangular form. Compute C and show that CA is an upper triangular matrix where A is the coefficient matrix.

8. Let

$$A = \begin{bmatrix} 1 & a & b \\ 0 & 1 & a \\ 0 & 0 & 1 \end{bmatrix}.$$

   Compute A^n.

9. Let $A \in \mathbb{R}^{n \times n}$ with entries

$$a_{ij} = \begin{cases} 1 & i = j - 1 \\ 0 & \text{otherwise.} \end{cases}$$

   Compute

   (a) A^2,

   (b) A^{n-1},

   (c) A^n.

10. Compute AB using the 4 different perspectives of matrix-matrix multiplication

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$