MAT 22A Problem Set 4 (Due 7/8 8 AM)

1. Compute the inverse of the following matrices using Gauss-Jordan elimination and the augmented matrix:

   (a) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 & 2 & -1 \\ 1 & 4 & 1 \\ -1 & 1 & 2 \end{bmatrix}$

2. Compute the LU decomposition for the following matrices

   (a) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$

   (b) $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 1 & 4 & 1 \end{bmatrix}$

3. Use the LU decomposition from the previous problem to solve the following systems

   (a) $A\mathbf{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

   (b) $A\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

   (c) $A\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

4. Use the inverse matrices found in problem 1 to find the solution to the following systems

   (a) $A\mathbf{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

   (b) $A\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

(c) $A\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

5. Show that $(AB)^{\mathrm{T}} = B^{\mathrm{T}} A^{\mathrm{T}}$ for the following matrices

   (a) $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 1 & 0 \\ 1 & 2 & 3 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 4 & 1 \\ -1 & 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

6. Let $A \in \mathbb{R}^{n \times n}$. Show that $A^{\mathrm{T}} A$ is a symmetric matrix.

7. Let $E_{ij} \in \mathbb{R}^{n \times n}$ be the elimination matrix

$$E_{ij} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ & \ddots & \ddots & \vdots \\ & -\ell_{ij} & \ddots & 0 \\ & & & 1 \end{bmatrix}$$

   where ℓ_{ij} is a multiplier in position (i, j). Show that

$$E_{ij}^{-1} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ & \ddots & \ddots & \vdots \\ & \ell_{ij} & \ddots & 0 \\ & & & 1 \end{bmatrix}$$

   where ℓ_{ij} is in position (i, j).