MAT 22A Problem Set 5 (Due 7/10 8 AM)

1. Let
$$P_n = \left\{ f : f = \sum_{\alpha=0}^{n} c_\alpha x^\alpha, \quad c_\alpha \in \mathbb{R} \right\}$$
be the set of polynomials of degree n. Show that P_n is a vector space.

2. Let V be the space of real-valued functions. A function is odd if $f(-x) = -f(x)$ and even if $f(-x) = f(x)$. Let W be the set of odd real-valued functions and X the set of even real valued functions. Is W a subspace of V? Is X a subspace of V?

3. Let V be the set of pairs (x, y) with $x, y \in \mathbb{R}$. Define
$$(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, 0)$$
$$c(x_1, y_1) = (cx_1, 0)$$
where $c \in \mathbb{R}$. Is V with these operations a vector space?

4. Show that the set of real diagonal matrices forms a vector space.

5. For the following matrices (a) Find the nullspace of A and (b) Find the rank of A.

   (i) $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \end{bmatrix}$

   (ii) $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \\ 1 & 0 \end{bmatrix}$

   (iii) $A = \begin{bmatrix} 1 & 2 & -1 & 0 & 1 & -1 \\ 1 & 2 & 2 & 0 & 1 & 0 \\ 0 & 0 & 2 & 3 & 2 & -1 \end{bmatrix}$