MAT 22A Problem Set 6 (Due 7/17 8 AM)

1. Find the complete solution to

$$\begin{bmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 4 & 8 \\ 0 & 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} -3 \\ -4 \\ 2 \end{bmatrix}.$$

   Write the complete solution as $\mathbf{x} = \mathbf{x}_p + \mathbf{x}_n$.

2. Give an example of a matrix A for each of the four possibilities for linear equations depending on the rank r. Show that the matrix has the corresponding number of solutions.

3. Find a basis for the column space and nullspace of A. What is the dimension of the nullspace and columnspace? What is the rank of A?

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 3 & 1 \\ 3 & 1 & -1 \end{bmatrix}.$$

4. Suppose $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$ are linearly independent.

   (a) Let $\mathbf{v}_1 = \mathbf{u}_2 - \mathbf{u}_3$, $\mathbf{v}_2 = \mathbf{u}_1 - \mathbf{u}_3$, and $\mathbf{v}_3 = \mathbf{u}_1 - \mathbf{u}_2$. Determine whether $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ are linearly independent.

   (b) Let $\mathbf{v}_1 = \mathbf{u}_2 + \mathbf{u}_3$, $\mathbf{v}_2 = \mathbf{u}_1 + \mathbf{u}_3$, and $\mathbf{v}_3 = \mathbf{u}_1 + \mathbf{u}_2$. Determine whether $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ are linearly independent.