

1. Let $S = \text{span} \left(\begin{bmatrix} 1 \\ 2 \\ 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -1 \\ 2 \\ 1 \end{bmatrix} \right)$. Find a basis for S^\perp and determine the dimension of S^\perp .
2. Find the nullspace of an $n \times n$ invertible matrix A and determine the dimension of $N(A)$. Explain your reasoning.
3. Find the pseudoinverse of the following matrix, and compute the product of the pseudoinverse and given matrix.

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 2 \\ 2 & 2 & -1 \\ -1 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$$

4. Let S be a subspace of the vector space \mathbb{R}^n . Prove that $\mathbb{R}^n = S \oplus S^\perp$. That is, for any $\mathbf{v} \in \mathbb{R}^n$,

$$\mathbf{v} = \mathbf{s} + \mathbf{t}$$

where $\mathbf{s} \in S$ and $\mathbf{t} \in S^\perp$.