MAT 22A Problem Set 9 (Due 7/24 5 PM)

1. Find the projection of matrix P and projection $\mathbf{p}$ of $\mathbf{b} = \begin{bmatrix} 5 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ onto the subspace spanned by $\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix} \right\}$.

   What is the length of the error vector?

2. Prove the following properties of the projection matrix P:

   (a) $P^2 = P$

   (b) $P^{\mathrm{T}} = P$

3. Let S_1 and S_2 be subspaces of $\mathbb{R}^4$. S_1 is spanned by $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$ and S_2 spanned by $\left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix} \right\}$. Show that S_1 and S_2 are orthogonal subspaces. Find the orthogonal complement of $S_1 + S_2 = \operatorname{span}\left(\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix} \right\} \right)$.

   Let P_1 be the projection matrix onto S_1, P_2 the projection matrix onto S_2 and P_3 the projection matrix onto $(S_1 + S_2)^{\perp}$. Show that $P_1 + P_2 + P_3 = I$.

4. Consider the following data points: $(-2, -6)$, $(-1, 0)$, $(0, 0)$, $(1, 0)$, and $(2, 6)$. Interpolate the data using the following polynomials:

   (a) Line

   (b) Parabola

   (c) Cubic

   Which is the best fit to the data? Why?