MAT 22B Group Work 3 (Due 7/9 11:59 PM)

The goal of this assignment is to use Euler's method and compare the numerical method to the exact solution.

1. Consider the following initial value problem

$$y' = 3\cos(t) - 2y, \quad y(0) = 0.$$

   (a) By hand, find the exact solution of the initial value problem.

   (b) Implement Euler's method to solve the initial value problem for $t \in [0, 4\pi]$.

   (c) Let $n = 2^k$ and $h = \frac{4\pi}{n-1}$. For $k = 4, 7, 10, 14$, plot the numerical solution and exact solution. MATLAB's subplot[1] allows multiple plots to be displayed in a single figure.

2. Let $n = 2^k$ and $h = \frac{4\pi}{n-1}$. For $k = 4, \ldots, 14$, compute the error

$$\|e_k\|_2 = \|\tilde{y} - y\|_2 = \sqrt{\sum_{i=1}^{n} |\tilde{y}_i - y_i|^2}$$

   where $\tilde{y}$ is the numerical solution and y is the exact solution. You can use MATLAB's norm[2] function. Produce a loglog[3] plot of the error. What is the slope of the line in the loglog plot?

3. (Bonus) Repeat problems 1 and 2 using the implicit Euler's method. How does the error of the two methods compare?

[1] https://www.mathworks.com/help/matlab/ref/subplot.html
[2] https://www.mathworks.com/help/matlab/ref/norm.html
[3] https://www.mathworks.com/help/matlab/ref/loglog.html