

MAT 22B Application 2 (Due 7/6 11:59 PM)

The goal of this assignment is to use numerical integration and numerical root solving functions to explicitly compute the solution of a differential equation.

1. Consider the following initial value problem

$$\frac{dy}{dt} + 2ty = \tan(t), \quad y(0) = 50.$$

Solve the initial value problem and plot the solution for $t \in [0, t^*)$ where t^* is the largest possible value for which the solution of the initial value problem is valid. Submit a plot of the solution. **Hint:** Find a solution involving a definite integral, and use MATLAB's `integral`¹ function.

2. Consider the following initial value problem

$$\frac{dy}{dx} = \frac{xe^x}{3y^2 - 1}, \quad y(0) = 0.$$

Solve the initial value problem and plot the solution for $x \in [0, 2]$. Give a reason for the observed jump discontinuity of the solution, determine the interval of validity, and submit a plot of the solution. **Hint:** First, find an implicit expression for the solution in the form $F(x, y) = 0$. Next, for each $x_i \in [0, 2]$, find y_i so that $F(x_i, y_i) = 0$ using MATLAB's `fzero`² function. For the guess required by `fzero`, use y_{i-1} (the previous value that you found).

¹<https://www.mathworks.com/help/matlab/ref/integral.html>

²<https://www.mathworks.com/help/matlab/ref/fzero.html>