MAT 22B Application 3 (Due 7/15 11:59 PM)

The goal of this assignment is to use Euler's method and compare the numerical method to the exact solution.

1. Consider the following initial value problem

$$y' = 3\cos(t) - 2y, \quad y(0) = 0.$$

   (a) By hand, find the exact solution to the initial value problem.

   (b) Implement Euler's method to solve the initial value problem for $t \in [0, 4\pi]$.

   (c) Let $n = 2^k$ and $h = \frac{4\pi}{n-1}$. For $k = 4, 7, 10, 14$, plot the numerical solution and exact solution. MATLAB's subplot[1] allows multiple plots to be displayed in a single figure.

   (d) Discuss what you observe about different values of k.

2. Over 10 species of trout are native to California[2], but with the changing climate, warmer waters and decreased flows have impacted their numbers. The state of California is home to the oldest fish hatchery west of the Mississippi, and hatcheries across the state[3] play a vital role in maintaining our freshwater fisheries. Around the end of World War II, the Department of Fish and Game (today, the Department of Fish and Wildlife) began experimenting with stocking remote alpine lakes with an aerial approach. Let us use our knowledge of air resistance and Euler's method to develop a computer program to help stock these lakes.

   (a) Stocking trout from the air requires knowing where the trout will end up. Write a program which inputs the plane's position and velocity and predicts the location of the skydiving fish. Be sure to state any assumptions or simplifications you make.

   (b) Suppose the lake lies in the valley of the function

$$f(x) = 1.5e^{-\frac{(x-3)^2(x+3)^2}{500}},$$

   where distances are in miles, and suppose you are flying in a Cessna 185 Skywagon which has a stall speed of 56 mph. Determine a safe speed of approach and altitude which will allow you to drop fish for the longest period possible.

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[1]https://www.mathworks.com/help/matlab/ref/subplot.html
[2]https://wildlife.ca.gov/Fishing/Inland
[3]https://wildlife.ca.gov/Fishing/Hatcheries