MAT 22B Midterm (Summer Session I 2021)

1. A chemical company is creating a brine solution in a 100 gallon cylindrical tank with an open top. A brine solution with a concentration of 5 grams per gallon is pumped into the tank at a rate of 10 gallons per minute. The tank has an outflow port through which the solution in the tank flows out at a rate of 10 gallons per minute. The tank initially contains 50 gallons of pure water.

   (a) Set up an initial value problem to determine the concentration of the solution in the tank at time t. You may set up multiple initial value problems for different quantities. (5 points)

   (b) Solve the initial value problem. (5 points)

   (c) What is the concentration of the solution in the tank in the long run? (5 points)

2. Consider the following initial value problem

$$\frac{dy}{dt} + 2y = \tan t, \quad y(0) = 2.$$

   (a) Determine the interval of validity. (5 points)

   (b) Determine the solution to the IVP. You may leave any difficult integrals unsolved, but do not forget to use the initial condition. (5 points)

3. Consider the following initial value problem

$$\frac{dy}{dt} = y(1 - y), \quad y(0) = 0.$$

   (a) Find and classify equilibria. (5 points)

   (b) Plot $f(y)$ vs y and draw the phase line. (5 points)

   (c) Where does a unique solution to the initial value problem exist? Justify your answer. (5 points)

   (d) Solve the initial value problem. (5 points)

4. Consider the following initial value problem

$$y - x^3 + (x + y^3)y' = 0, \quad y(0) = 1.$$

   (a) Show that the differential equation is exact. (5 points)

   (b) Solve the initial value problem. You may leave the solution in implicit form. (10 points)

5. Consider the following initial value problem

$$\frac{dy}{dt} = y + t, \quad y(0) = 0.$$

   (a) Where does a unique solution to the initial value problem exist? Justify your answer. (5 points)

   (b) Write down the Euler iteration. (5 points)

   (c) Let $\phi_0(t) = 0$. Determine the n^{th} Picard Iterate, $\phi_n(t)$. (5 points)

6. Consider the following initial value problem

$$y'' + 2y' + 2y = 0, \quad y(0) = 0, \quad y'(0) = 1.$$

   (a) Find two solutions and show that they form a fundamental set of solutions. (10 points)

   (b) Solve the initial value problem. (10 points)

   (c) Determine the long-term behavior of the solution. (5 points)