MAT 22B Problem Set 6 (Due 7/22)

1. Solve the following initial value problems.

   (a) $y'' + 4y = t^2 + 3e^t$, $\quad y(0) = 0$, $\quad y'(0) = 2$

   (b) $y'' - 2y' + y = te^t + 4$, $\quad y(0) = 2$, $\quad y'(0) = -1$

   (c) $y'' + 3y' + 2y = e^t(t^2 + 1)\sin(2t) + 3e^{-t}\cos(t) + 4e^t$, $\quad y(0) = 0$, $\quad y'(0) = 0$

2. Solve the following differential equation

$$y'' + y = \begin{cases} t & 0 \le t \le \pi \\ \pi e^{\pi - t} & t > \pi \end{cases}$$

   with initial conditions $y(0) = 0$ and $y'(0) = 1$, and assume that y and y' are continuous at $t = \pi$.

3. Find the general solution of the following differential equations and determine the interval of validity.

   (a) $y'' + y = \tan(t)$

   (b) $y'' + 4y' + 4y = t^{-2}e^{-2t}$

   (c) $y'' - 2y' + y = \frac{e^t}{1+t^2}$

4. Show that the solution to the initial value problem

$$y'' + by' + cy = g(t), \quad y(t_0) = 0, \quad y'(t_0) = 0$$

   may be written in the form

$$y(t) = \int_{t_0}^{t} K(t - s)g(s)\, ds.$$

5. A spring is streched 10 cm by a force of 3 N. A mass of 2 kg is hung from the spring and is also attached to a viscous damped that exerts a force of 3 N when the velocity of the mass is 5 meters per second. If the mass is pulled down 5 cm below its equilibrium position and given an initial downward velocity of 10 cm per second, determine its position u at any time t. Find the quasi-frequency μ and the ratio of μ to the natural frequency of the corresponding undamped motion.

6. A mass of 5 kg stretches a spring 10 cm. The mass is acted on by an external force of $10\sin\left(\frac{t}{2}\right)$ N and moves in a madium that imparts a viscous force of 2 N when the speed of the mass is 4 cm per second. If the mass is set in motion from its equilibrium position with an initial velocity of 3 cm per second, formulate the initial value problem describing the motion of the mass.

7. Find the solution of the initial value problem

$$u'' + u = F(t), \quad u(0) = 0, \quad u'(0) = 0,$$

   where

$$F(t) = \begin{cases} F_0 t, & 0 \le t \le \pi \\ F_0(2\pi - t), & \pi < t \le 2\pi, \\ 0, & 2\pi < t. \end{cases}$$