MAT 22B Problem Set 7 (Due 7/27)

1. Compute the Laplace transform of the following functions

   (a) $f(t) = \begin{cases} 1 & 0 \le t < \pi \\ 0 & \pi \le t < \infty \end{cases}$

   (b) $f(t) = \begin{cases} t & 0 \le t < 1 \\ 2 - t & 1 \le t < 2 \\ 0 & 2 \le t < \infty \end{cases}$

   (c) $f(t) = t^n e^{at}$

2. Compute the inverse Laplace transform of the following functions

   (a) $F(s) = \frac{4}{(s-1)^3}$

   (b) $F(s) = \frac{2s+2}{s^2+2s+5}$

   (c) $F(s) = \frac{8s^2-4s+12}{s(s^2+4)}$

3. Use the Laplace Transform to solve the following initial value problems

   (a) $y'' - y' - 6y = 0, \quad y(0) = 1, \quad y'(0) = -1$

   (b) $y'' - 2y' + 2y = 0, \quad y(0) = 0, \quad y'(0) = 1$

   (c) $y^{(4)} - y = 0, \quad y(0) = 1, \quad y'(0) = 0, \quad y''(0) = 1, \quad y'''(0) = 0$

   (d) $y'' - 2y' + 2y = e^{-t}, \quad y(0) = 0, \quad y'(0) = 1$

4. Transform the following differential equations into a system of first-order differential equations. If initial values are given, also transform the initial values of the original equation to appropriate initial conditions for the system of equations.

   (a) $t^2u'' + tu' + (t^2 - 0.25)u = 0$

   (b) $u^{(4)} - u = 0$

   (c) $u'' + 0.25u' + 4u = 2\cos(3t), \quad u(0) = 1, \quad u'(0) = -2$

   (d) $u'' + p(t)u' + q(t)u = g(t), \quad u(0) = u_0, \quad u'(0) = u_0'$

5. Determine the general solution of the following system

$$\mathbf{x}' = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix} \mathbf{x}.$$

6. Determine the general solution in terms of real-valued functions

$$\mathbf{x}' = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{pmatrix} \mathbf{x}.$$

7. Find the fundamental matrix of

$$\mathbf{x}' = \begin{pmatrix} 1 & -1 \\ 5 & -3 \end{pmatrix} \mathbf{x}.$$