MAT 22B Application 4 (Due 7/31 11:59 PM)

The goal of this assignment is play around with the logistic equation. Consider the logistic difference equation

\[ u_{n+1} = \rho u_n (1 - u_n) . \]

We will explore the behavior of solutions for different values of \( \rho \).

1. Plot \( u_n \) vs \( n \) for the following values of \( \rho \). Use at least 4 different initial values of \( u_0 \in (0,1) \) for each value of \( \rho \).
   
   (i) \( \rho = 0.8 \)
   (ii) \( \rho = 1.5 \)
   (iii) \( \rho = 2.8 \)
   (iv) \( \rho = 3.2 \)
   (v) \( \rho = 3.5 \)
   (vi) \( \rho = 3.57 \)
   (vii) \( \rho = 3.6 \)
   (viii) \( \rho = 3.7 \)
   (ix) \( \rho = 3.9 \)
   (x) \( \rho = 4 \)

2. Create a cobwebbing diagram for the logistic difference equation for the following values of \( \rho \). Use at least 4 different initial values of \( u_0 \in (0,1) \) for each value of \( \rho \).

   (i) \( \rho = 0.8 \)
   (ii) \( \rho = 1.5 \)
   (iii) \( \rho = 2.8 \)
   (iv) \( \rho = 3.2 \)
   (v) \( \rho = 3.5 \)
   (vi) \( \rho = 3.57 \)
   (vii) \( \rho = 3.6 \)
   (viii) \( \rho = 3.7 \)
   (ix) \( \rho = 3.9 \)
   (x) \( \rho = 4 \)

3. What do you observe about the behavior of solutions for the different values of \( \rho \)?

4. For a fixed value of \( \rho \), what do you observe about the behavior of solutions? If they start close, do they remain close? Is this true for all values of \( \rho \)?