MAT 22B Application 5 (Due 9/9 11:59 PM)

The goal of this assignment is to explore damped and undamped oscillations and periodic forcing.

1. Recall the mechanical spring system whose motion is governed by the differential equation

$$mu''(t) + \gamma u'(t) + ku(t) = F(t).$$

Here, m is the mass of the block attached to the spring, γ is the damping constant, k is the spring constant, u is the displacement from the spring's equilibrium position, and $F(t)$ is an applied external force.

   (a) Solve the second-order differential equation analytically.

   (b) Rewrite the differential equation as a system of first-order equations.

   (c) Solve the system of first-order equations analytically.

   (d) Write a program which uses Euler's method to solve the system of first-order equations numerically.

   (e) Plot each of the 4 solutions you have found. Explain how the plots are related to one another and how some may look different but describe the same physical process.

2. Now, let us explore some qualitative properties of the spring-mass system. Consider a simple car suspension system which is composed of a spring and a damping mechanism for the spring. The spring hides the bumps and holes in the road from the driver, and the damping system prevents the body of the car from experiencing excessively long cycles of bobbing up and down[1].

   (a) When $\gamma = 0$, the system undergoes undamped oscillations. Plot a solution which demonstrates undamped oscillations, describe the qualitative features of the solution, and describe how this would feel when riding in the car.

   (b) When $0 < \gamma < 2\sqrt{km}$, the system experiences damped oscillations. For 4 different values of γ, plot the solutions in a single figure, describe the qualitative features of the solutions, and describe how different values of γ would feel when riding in the car.

   (c) When $\gamma = 2\sqrt{km}$, the system is called critically damped. Plot a solution which is critically damped, describe its qualitative features, and describe how this would feel when riding in the car.

   (d) When $\gamma > 2\sqrt{km}$, the system is called overdamped. For 4 different values of γ, plot the solutions in a single figure, describe the qualitative features of the solutions, and describe how different values of γ would feel when riding in the car.

   (e) Plot an undamped, a damped, a critically damped, and an overdamped solution in a single figure. Describe how you can visually identify which solution corresponds to which solution.

   (f) Now, consider the case when $\gamma = 0$. Suppose you are driving along a road with bumps that are placed so that the car experiences a periodic applied force of $F(t) = \cos\left(\sqrt{\frac{k}{m}}t\right)$[2]. Describe your experience riding in the car.

[1] https://youtu.be/PFaLUwLs1IM?t=71

[2] https://youtu.be/Ef93WmlEho0