MAT 22B Final Topics

Below is a list of topics that will be covered on the Final. The list shows the major topics that we have covered in lecture, but it may not be an exhaustive list. The final will be cumulative, but there will be an emphasis on the material covered after the midterm.

1. Introduction to Differential Equations

   - Building models using differential equations
   - Direction fields and solution trajectories
   - Solution of a differential equation and the initial value problem
   - Classification of differential equations

2. First Order Differential Equations

   - First order linear equations, method of integrating factors
   - Separable differential equations
   - Modeling and analysis of models
   - Implicit solutions and solutions involving integrals
   - Linear vs Nonlinear differential equations
   - Existence and uniqueness theorems
   - Interval of existence
   - Autonomous differential equations
   - Exponential and logistic growth
   - Population dynamics
   - Equilibria, classification, and the phase line
   - Euler's method
   - Picard Iterates and convergence
   - Difference equations, equilibrium solutions, and long-term behavior

3. Second Order Linear Differential Equations

   - Homogeneous second order linear differential equations with constant coefficients: distinct, complex, and repeated roots
   - Initial value problem for second order differential equations
   - The Wronskian, the fundamental set of solutions, and the general solution
   - Abel's theorem
   - Euler's formula
   - Reduction of order
   - Solution of the nonhomogeneous equation
   - Method of undetermined coefficients
   - Variation of parameters
   - Mass-spring systems
   - Damped, undamped oscillations
   - Underdamped, overdamped, critically damped systems
   - Forced oscillations and resonance

4. Laplace Transform

   - Definition of the Laplace transform
   - Computing Laplace transform of functions

- Solving initial value problems using Laplace transform
- Determining inverse Laplace transform

5. Systems of First-Order Linear Equations

- Existence and uniqueness
- Abel's theorem for systems of ODEs
- Solving homogeneous linear systems with constant coefficients
  - Real and distinct eigenvalues
  - Complex eigenvalues
  - Repeated eigenvalues
- Fundamental matrices
- Matrix exponential
- Nonhomogeneous linear systems
- Undetermined coefficients
- Variation of parameters

Some advice:

- Review your notes and make sure you understand the topics and examples we have covered during lecture.

- Review the problem sets and make sure you understand how to do the problems without the solution keys.

- Practice the problems from the problem sets and make new problems by changing the numbers and/or size of the matrices in the problem sets. You can check your answers using MATLAB.

- Stop by office hours, and we can chat about any material you are struggling with, extra examples, or anything else!