MAT 22B Practice Midterm

August 18, 2022

Name: _____

Student ID: _____

- Carefully read each problem and make your arguments clear.

- You may not use any notes, books, or other outside materials.

- No electronic devices may be used to complete this exam.

- Work on your own exam. Any suspicions of violating the UC Davis Code of Academic Conduct will be reported to the Office of Student Support and Judicial Affairs.

Question	Points	Score
1	15	
2	10	
3	25	
4	10	
5	15	
6	25	
Total:	100	

1. A chemical company is creating a brine solution in a 100 gallon cylindrical tank with an open top. A brine solution with a concentration of 5 grams per gallon is pumped into the tank at a rate of 10 gallons per minute. The tank has an outflow port through which the solution in the tank flows out at a rate of 10 gallons per minute. The tank initially contains 50 gallons of pure water.

   (a) (5 points) Set up an initial value problem to determine the concentration of the solution in the tank at time t.

   (b) (5 points) Solve the initial value problem.

   (c) (5 points) What is the concentration of the solution in the tank in the long run?

2. Consider the following initial value problem

$$\frac{dy}{dt} + 2y = \frac{1}{t^2 - 4t + 3}, \quad y(2) = 1.$$

(a) (5 points) Determine the interval of validity. (5 points)

(b) (5 points) Determine the solution to the IVP. You may leave any difficult or tedious integrals unsolved, but do not forget to use the initial condition. (5 points)

3. Consider the following initial value problem

$$\frac{dy}{dt} = y(y^2 - 4), \quad y(0) = 1.$$

(a) (10 points) Find and classify equilibria.

(b) (10 points) Plot $f(y)$ vs y and draw the phase line.

(c) (5 points) Determine the long-time behavior of the solution to the initial value problem.

4. Consider the following initial value problem

$$t^2 y'' + 4ty' + 2y = 0, \quad y(1) = 1, \quad y'(1) = 0.$$

(a) (5 points) Find the general solution. (Hint: Consider the ansatz $y(t) = t^r$.)

(b) (5 points) Determine the solution to the initial value problem.

5. Consider the following initial value problem

$$\frac{dy}{dt} = y + t, \quad y(0) = 0.$$

(a) (5 points) Where does a unique solution to the initial value problem exist? Justify your answer.

(b) (5 points) Write down the Euler iteration.

(c) (5 points) Let $\phi_0(t) = 0$. Determine the nth Picard Iterate, $\phi_n(t)$.

6. Consider the following initial value problem

$$y'' + 4y' + 5y = 0, \quad y(0) = 0, \quad y'(0) = 1.$$

(a) (10 points) Find two solutions and show that they form a fundamental set of solutions.

(b) (10 points) Solve the initial value problem.

(c) (5 points) Determine the long-term behavior of the solution.