MAT 22B Problem Set 1 (Due 8/5)

1. For the following differential equations, plot the direction field, draw some integral curves, and determine what happens to y as $t \to \infty$.
   (a) $y' = y(y-4)$
   (b) $y' = e^{-t} + y$.
2. The California Delta[1] is fed by the Sacramento and San Joaqiun rivers. The rivers provide a source of freshwater to the region, and lowered flows from the Sacramento and San Joaquin result in salinity intrusion. Create a model of the salinity in the California Delta, set up an appropriate initial value problem, and solve the initial value problem.
3. Suppose an ice cream sandwich melts at a rate proportional to its surface area. Determine a differential equation for the volume of ice cream at some time t. When will the ice cream be completely melted?
4. Solve the following initial value problem
$$\frac{dy}{dt} = ay + b, \quad y(0) = y_0$$
where a and b are arbitrary constants.
5. Classify the following differential equations.
   (a) $\frac{d^3y}{dt^3} + \sin(t)\frac{dy}{dt} = \cosh(t)$
   (b) $\frac{\partial^2 u}{\partial x \partial y} + \cos(x) = 0$
   (c) $\frac{\partial^2 u}{\partial x^2} + \left(\frac{\partial u}{\partial y}\right)^2 + \sin(x) = 0.$
   (d) $y^{(4)} + \sin(y) = t^2$
6. Determine whether the given solutions are solutions of the differential equation.
   (a) $y'' + y' - 2y = 0$, $y_1(t) = e^t$, $y_2(t) = e^{-2t}$, $y_3 = c_1y_1(t) + c_2y_2(t)$ where c_1 and c_2 are arbitrary constants
   (b) $t^2y'' - 2y = 0$, $y_1(t) = \frac{1}{t}$, $y_2(t) = t^2$, $y_3 = c_1y_1(t) + c_2y_2(t)$ where c_1 and c_2 are arbitrary constants
   (c) $a^2u_{xx} = u_{tt}$, $u(x,t) = \sin(\lambda x)\cos(a\lambda t)$ where λ and a are constants
7. Consider the ansatz $y(t) = e^{rt}$. Find r so that y is a solution to the following differential equations. Note that a and b are arbitrary constants. Be careful of special cases.
   (a) $y' + ay = 0$
   (b) $y'' + ay' + by = 0$
   (c) $y^{(4)} + ay'' + by = 0$
8. Consider the ansatz $y(t) = t^r$. Find r so that y is a solution to the following differential equations for $t > 0$. Note that a and b are arbitrary constants. Be careful of special cases.
   (a) $ty' + ay = 0.$
   (b) $t^2y'' + aty' + by = 0.$

[1] https://en.wikipedia.org/wiki/Sacramento-San_Joaquin_River_Delta