MAT 22B Problem Set 2 (Due 8/12)

1. Solve the ODE, and determine the behavior of solutions as $t \to \infty$.

   (a) $y' - 2y = 3e^t$

   (b) $y' + \frac{1}{t}y = 3\cos(2t)$

   (c) $2y' + y = 3t^2$

2. Consider the general linear first order equation

$$\frac{dy}{dt} + p(t)y = g(t).$$

   (a) Determine the solution when $g(t) = 0$ for all t.

   (b) If $g(t) \neq 0$ for all t, suppose that

$$y = c(t)e^{-\int p(t)\, dt}$$

   is the solution where $c(t)$ is an unknown function. Find a differential equation that $c(t)$ must satisfy.

   (c) Solve the differential equation you found in part (b) and use your result to determine the solution to the general first order linear equation. Verify the solution you found satisfies the differential equation.

   (d) Use this technique to solve

$$y' - 2y = t^2e^{2t}.$$

3. Solve the ODE and determine the interval of validity.

   (a) $y' = (1 - 2x)y^2$

   (b) $y' = \frac{2x}{1+2y}$

   (c) $y' = \frac{3x^2}{3y^2-4}$

4. A function $f(x, y)$ is called homogeneous of degree n if

$$f(tx, ty) = t^n f(x, y).$$

   Consider the differential equation

$$\frac{dy}{dx} = f(x, y)$$

   and suppose that f is homogeneous of degree zero.

   (a) Let $t = \frac{1}{x}$ and show that $f(x, y) = f(1, z)$ where $z = \frac{y}{x}$.

   (b) Use the subsitution $z = \frac{y}{x}$ to transform the given differential equation into a differential equation involving only x and z.

   (c) Show that the resulting differential equation is separable.

   (d) Use this technique to solve

$$\frac{dy}{dx} = \frac{x^2 - 2y^2}{xy}.$$

   Note: Be sure to show that the RHS is indeed homogeneous of degree zero.

5. The Tulare Lake[1] was the largest freshwater lake west of the Mississippi River and was located in the southern end of the San Joaquin Valley. The Tulare Lake was fed by the Kern, Kaweah, and Tule rivers, but the lake eventually disappeared due to diversions of its tributaries for agriculture and municipal uses by the end of the 19th century. The lake emptied into the San Joaquin river when the lake reached an elevation of 207 to 210 feet. Create a model of the water present in the Tulare Lake, specify an initial condition, and solve the initial value problem.

6. The Salton Sea[2] is a body of water located east of San Diego in the Colorado Desert, and it was once considered a resort destination. The Salton Sea has a high concentration of salts and pollution from agricultural runoff. The Alamo, Whitewater, and New river feed the Salton Sea, and the New river is one of the most polluted rivers in the United States. Assume that the Whitewater and Alamo rivers are clean, and the New river introduces pollutants into the Salton Sea. The Salton Sea has no outflow, but it is susceptible to evaporation. Create a model for the concentration of pollutants in the Salton Sea, specify an initial condition, and solve the initial value problem.

[1] https://en.wikipedia.org/wiki/Tulare_Lake
[2] https://en.wikipedia.org/wiki/Salton_Sea